21-301 Combinatorics
Homework 2
Due: Friday, September 12

1. Use induction to show that

$$\binom{n-1}{k} = \binom{n}{k} - \binom{n}{k-1} + \cdots \pm \binom{n}{0}.$$

2. **(a)** Let $\mathcal{S}_k$ denote the collection of k-sets $\{1 \le i_1 < i_2 < \cdots < i_k \le m-2\} \subseteq [m]$ such that $i_{t+1} - i_t \ge 3$ for $1 \le t < k$. Show that

$$|\mathcal{S}_k| = \binom{m-2k}{k}.$$

   (b) How many of the 3^n sequences $x_1 x_2 \cdots x_n$, $x_i \in \{a, b, c\}$, $i = 1, 2, \ldots, n$ are there such that abc does not appear as a consecutive subsequence e.g. if $n = 6$ then we include $aabbcc$ in the count, but we exclude $aabcba$.

   [You should use Inclusion-Exclusion and expect to have your answer as a sum.]

3. Find an expression for the size of the set

$$\{(x_1, x_2 \ldots, x_m)\} \in Z^m : \; x_1 + x_2 + \cdots + x_m = n \; and \; a \le x_j \le b \; for \; j = 1, 2, \ldots, m\}.$$

   [You should use Inclusion-Exclusion and expect to have your answer as a sum.]