21-301 Combinatorics Homework 10 Due: Monday, December 1

- 1. How many ways are there to k-color an  $n \times n$  chessboard when n is odd. The group G is the usual 8 element group e, a, b, c, p, q, r, s.
- 2. How many ways are there to arrange 2 M's, 4 A's, 5 T's and 6 H's under the condition that any arrangement and its inverse are to be considered the same.
- 3. A necklace is made of 10 beads strung together in a cycle. Find the pattern inventory for the two colourings of the necklace when the group G is the dihedral group  $D_{10}$ .

The *dihedral* group  $D_n$  is the group of symmetries of a regular *n*-gon under rotations  $R_0, R_1, \ldots, R_{n-1}$  and reflections  $S_1, S_1, \ldots, S_n$ . Here, assuming *n* is even, the permutations are

- (i)  $e = R_0, R_1, \ldots, R_{n-1}$  where  $R_i$  is a rotation through  $i\pi/4$ ,
- (ii)  $S_1, S_1, S_2, \ldots, S_{n/2}$  where  $S_i$  is a rotation about an axis joining two opposite vertices,
- (iii)  $S_{n/2+1}, S_{n/2+2}, \ldots, S_n$  where  $S_{n/2+i}$  is a rotation about an axis joining the midpoints of two opposite edges.