21-301 Combinatorics Homework 1 Due: Friday, September 5

1. How many integral solutions of

 $x_1 + x_2 + x_3 + x_4 + x_5 = 100$

satisfy $x_1 \ge 3$, $x_2 \ge 10$, $x_3 \ge -3$, $x_4 \ge 6$ and $x_5 \ge 0$?

2. Prove the following equation:

$$\sum_{i=k}^{n} \binom{i}{k} \binom{n}{i} = \binom{n}{k} 2^{n-k}.$$

3. How many ways are there of placing k 1's and n - k 0's at the vertices of an n vertex polygon, so that every pair of 1's are separated by at least 2 0's?