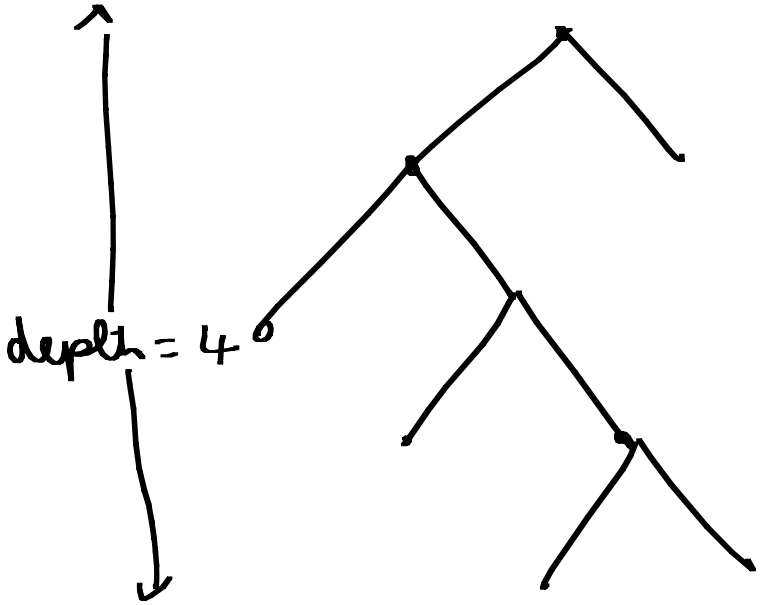
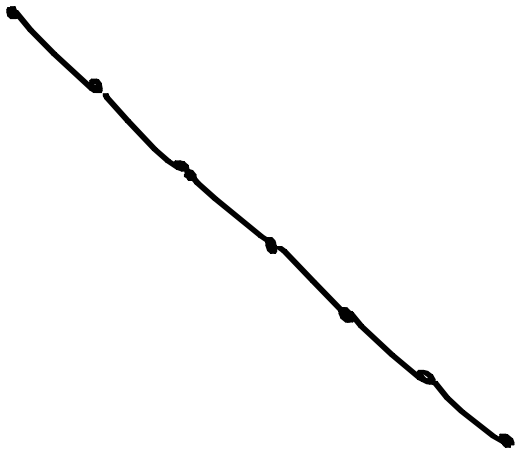
9/29/08

Binary Search Trees

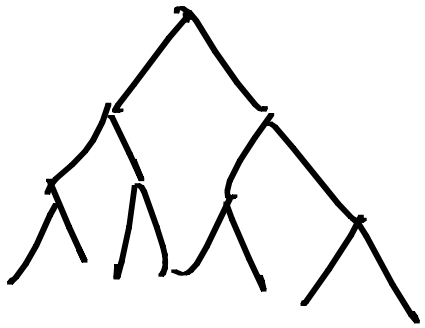


n nodes

Bad Tree

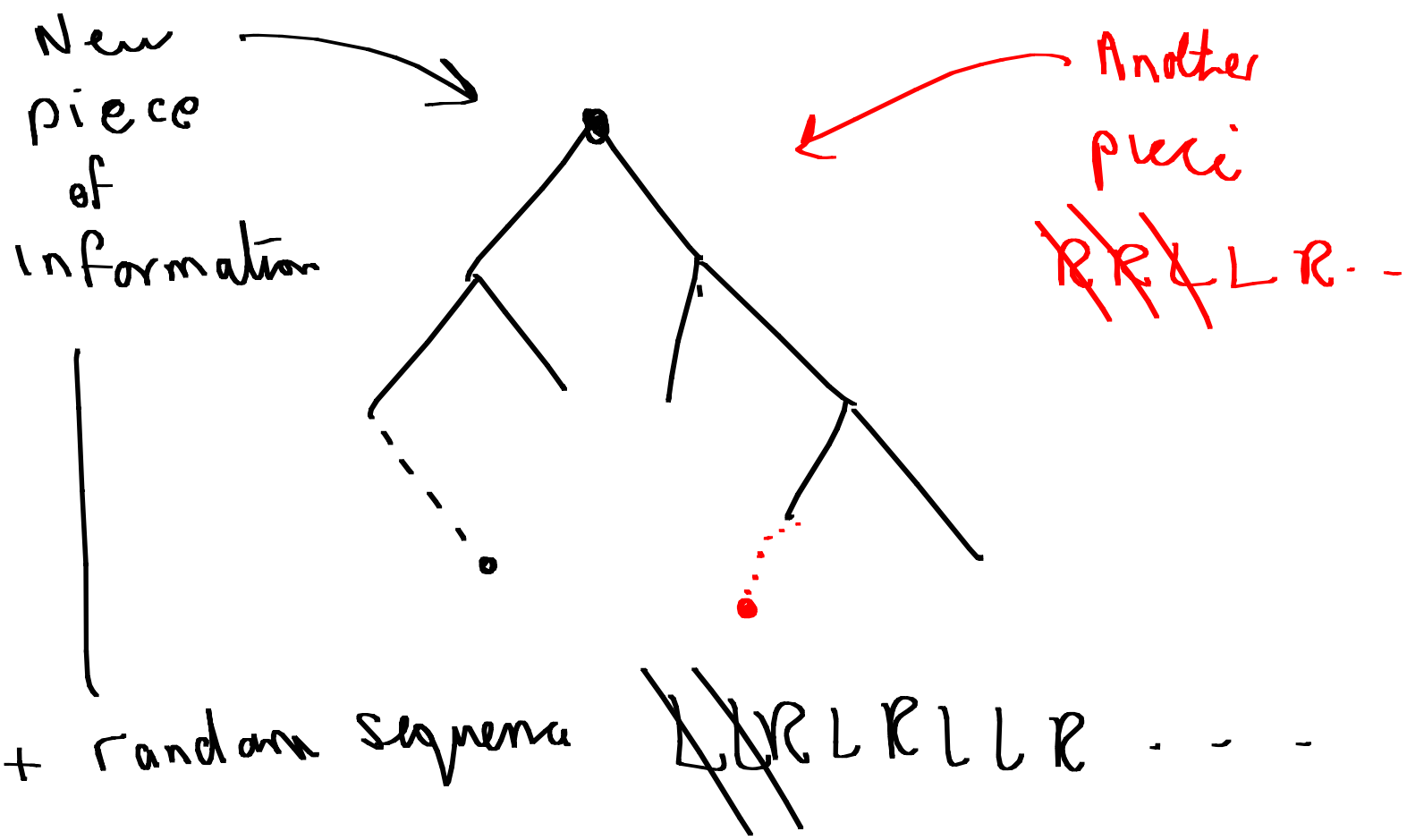


Good Tree



Good tree has depth $O(\log n)$

Suppose we build tree from random date.



Question: After adding n items, what is the likely depth of the tree?

D_n = depth of tree.

Random Variable.

Claim: $Pr(D_n \geq t) \leq \left(n 2^{-(t-1)/2}\right)^t$.

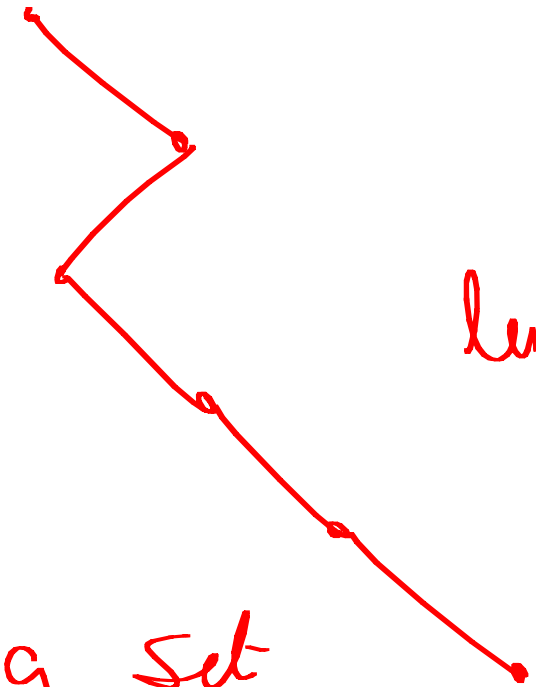
$\Omega = \{L, R\}^{n^2}$

LLRLLR - - - - -

Event: $DEEP = \{D_n \geq t\}$

Suppose DEEP occurs.

In the tree, there deep path P



length t

+ there is a set

S of t particles that create P

DEEP (P, S)

$$P(\text{DEEP}) = P\left[\bigcup_{P}\bigcup_{S} \text{DEEP}(P,S)\right]$$

$$\leq \sum_{P}\sum_{S} P[\text{DEEP}(P,S)]$$

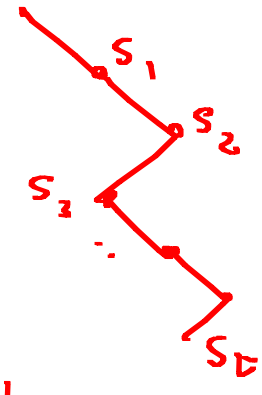
$$\leq \sum_{P}\sum_{S} \frac{1}{2^{t(t+1)/2}}$$

$$\leq (\#P) \times (\#S) \times \left(\frac{1}{2}\right)^{t(t+1)/2}$$

2^t $\binom{n}{t}$

$$\leq 2^t \cdot n^t \times \left(\frac{1}{2}\right)^{t(t+1)/2}$$

$S = \{s_1, s_2, \ldots, s_t\}$

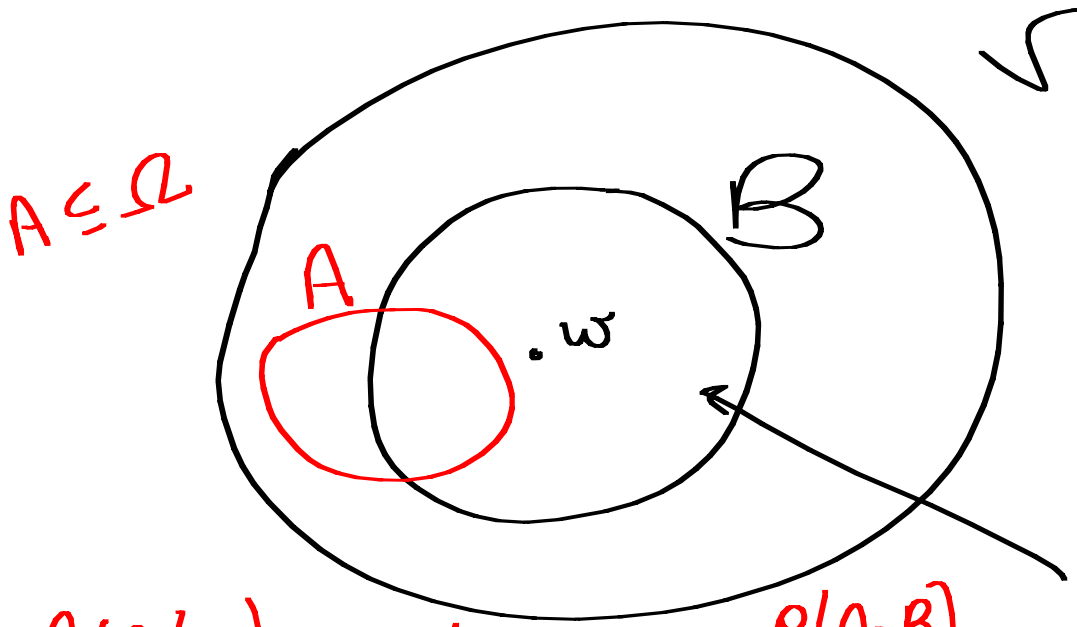


$$P_r(\quad) = \frac{1}{2} \times \frac{1}{2^2} \times \frac{1}{2^3} \times \cdots \times \frac{1}{2^t}$$

Conditional Probability

$P(A|B)$

Probability of A given B.



Create a new probability space P_B

$$P(A|B) = P_B(A \cap B) = \frac{P(A \cap B)}{P(B)}$$

$$\left(B, \frac{P(\omega)}{P(B)}\right)$$

Roll 2 dice (X_1, X_2)

$B = \{ X_1 \geq 3 \}$ [Slight different to slide]

$A = \{ X_1 + X_2 > 6 \}$

$$P(A|B) = \frac{P(A \cap B) = 1/2}{P(B) = 2/3} = \frac{3}{4}$$

$x_1 = 3 \quad x_2 = 4 \quad x_3 = 5 \quad x_4 = 6$

$$\frac{3 + 4 + 5 + 6}{36} = \frac{1}{2}$$

Family has 2 children

$\Omega = \{BB, BG, GB, GG\}$

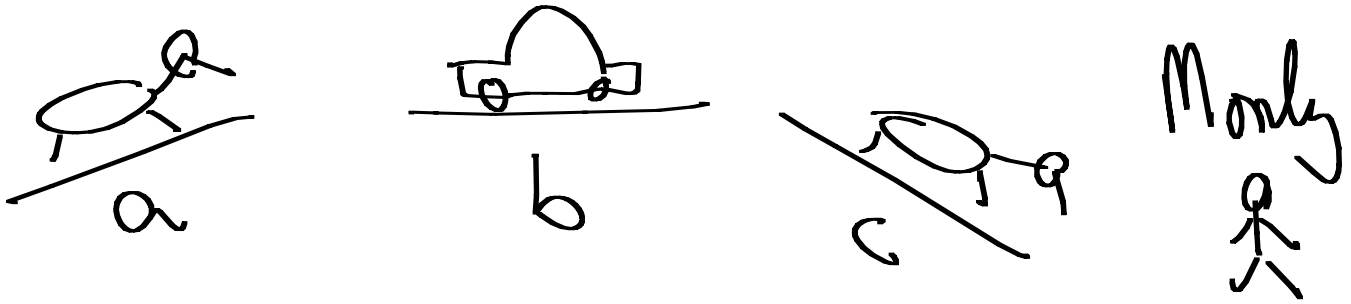
equally likely.

$B = \{$ at least one boy $\}$

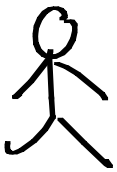
$A = \{$ two boys $\}$

$$P(A|B) = \frac{BB}{BG + GB + BB} = \frac{1}{3}$$

Monty Hall Paradox



contestant



Game: You choose a door, Monty says.
Monty opens another door, with a goat behind it. Monty gives you a chance to change your mind.

Answer: You should switch

P[other door has car] = $\frac{2}{3}$

$\frac{1}{3}$ a

$\frac{2}{3}$ b, c