9\19\08

$$u(x) = \frac{1}{1-3x}$$

x

$|x| < \frac{1}{3}$

$$= 1 + (3x) + (3x)^2 + \cdots + (3x)^n$$

$$= \sum_{n=0}^{\infty} 3^n x^n$$

$u_n \quad x^n$

$$u_n = 3^n.$$

$$a_n - a_{n-1} - a_{n-2} = 0 \qquad n \geq 2$$

$$\sum_{n \geq 2} \left(a_n - a_{n-1} - a_{n-2}\right) x^n = 0$$

$$\left[a(x) - a_0 - a_1 x\right] - \left[x\left(a(x) - a_0\right)\right] - x^2 a(x) = 0$$

$$a(x) = \frac{1}{1 - x - x^2}$$

$$a(x) = \frac{1}{1-x-x^2}$$

$$= \frac{1}{(\xi_1 - x)(\xi_2 - x)}$$

$$= \frac{A}{\xi_1 - x} + \frac{B}{\xi_2 - x}$$

$$= \frac{A/\xi_1}{1 - x/\xi_1} + \frac{B/\xi_2}{1 - x/\xi_2}$$

$$= \frac{A/\xi_1}{1 - x/\xi_1} + \frac{B/\xi_2}{1 - x/\xi_2}$$

$$= \frac{A}{\xi_1} \sum_{n=0}^{\infty} \frac{x^n}{\xi_1^n} + \frac{B}{\xi_2} \sum_{n=0}^{\infty} \frac{x^n}{\xi_2^n}$$

$$F_n = \frac{A}{\xi_1^{n+1}} + \frac{B}{\xi_2^{n+1}}$$

$$a_n - 6a_{n-1} + 9a_{n-2} = n \quad , n \geq 2$$

$$a_0 = 1, \quad a_2 = 2$$

$$\sum_{n\geq 2} \left(a_n - 6a_{n-1} + 9a_{n-2}\right) x^n = \sum_{n \geq 2} n x^n$$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots \qquad \frac{x}{(1-x)^2} - x$$

$$\frac{x}{(1-x)^2} = \left(1 + 2x + 3x^2 + \cdots + nx^{n-1} + \cdots\right) \times x$$

$$\sum_{n\geq 2}\left(a_n - 6a_{n-1} + 9a_{n-2}\right)x^n = \frac{x}{(1-x)^2} - x$$

$$\left[a(x) - a_0 - a_1 x\right] - 6x\left[a(x) - a_0\right] + 9x^2 a(x)$$

$$a(x) - 1 - 2x - 6x\left(a(x) - 1\right) + 9x^2 a(x) = \frac{x}{(1-x)^2} - x$$

$$a(x)\left[1 - 6x + 9x^2\right] = \frac{x}{(1-x)^2} - x + 1 + 2x - 6x$$

$$= \frac{x}{(1-x)^2} + 1 - 5x$$

$$a(x) = \frac{x}{(1-3x)^2(1-x)^2} + \frac{1}{(1-3x)^2} - \frac{5x}{(1-3x)^2}$$

$$= \frac{A}{1-x} + \frac{B}{(1-x)^2} + \frac{C}{1-3x} + \frac{D}{(1-3x)^2}$$

$$x + (1-x)^2 - 5x(1-x)^2$$

$$= A(1-x)(1-3x)^2 + B(1-3x)^2$$

$$+ C(1-x)^2(1-3x) + D(1-x)^2 \quad \forall x$$

$x=1$: $1 = 4B$;

$x=0$: $1 = A+B+C+D$

$x = 1/3$: $\frac{1}{3} + \frac{4}{9} - \frac{5}{3} \cdot \frac{4}{9} = \frac{4}{9} D$;

$$= \frac{A}{1-x} + \frac{B}{(1-x)^2} + \frac{C}{1-3x} + \frac{D}{(1-3x)^2}$$

$$= A\sum_{n=0}^{\infty} x^n + B\sum_{n=0}^{\infty}(n+1)x^n + C\sum_{n=0}^{n} 3^n x^n + D\sum_{n=0}^{\infty}(n+1)3^n x^n$$

$$a_n = A + (n+1)B + 3^n C + (n+1)3^n D$$

Product of generating functions

$$a(x) = \sum a_n x^n$$

$$b(x) = \sum b_n x^n$$

$$c(x) = a(x) \times b(x)$$

$$= \sum_{n=0}^{\infty} c_n x^n$$

$$c_n = \sum_{k=0}^{n} a_k b_{n-k}$$

$$(a_0 + a_1 x + a_2 x^2 + \dots) \times$$

$$(b_0 + b_1 x + b_2 x^2 + \dots)$$

$$= a_0 b_0 + (a_0 b_1 + a_1 b_0) x$$

$$+ (a_0 b_2 + a_1 b_1 + a_2 b_0) x^2$$

$$+ \dots$$