9\17\ 2008

a_n = # of objects of some type.

We want to know what a_n =

Unknown sequence $a_0, a_1, a_2, \ldots, a_n, \ldots$

Sometimes we can find equation

$$a_n = \text{function}(a_{n-1}, a_{n-2}, \ldots)$$

and solve.

b_n = # sequences in $\{a, b, c\}^n$ that do not contain aa as a consecutive subsequence.

$b_1 = 3$

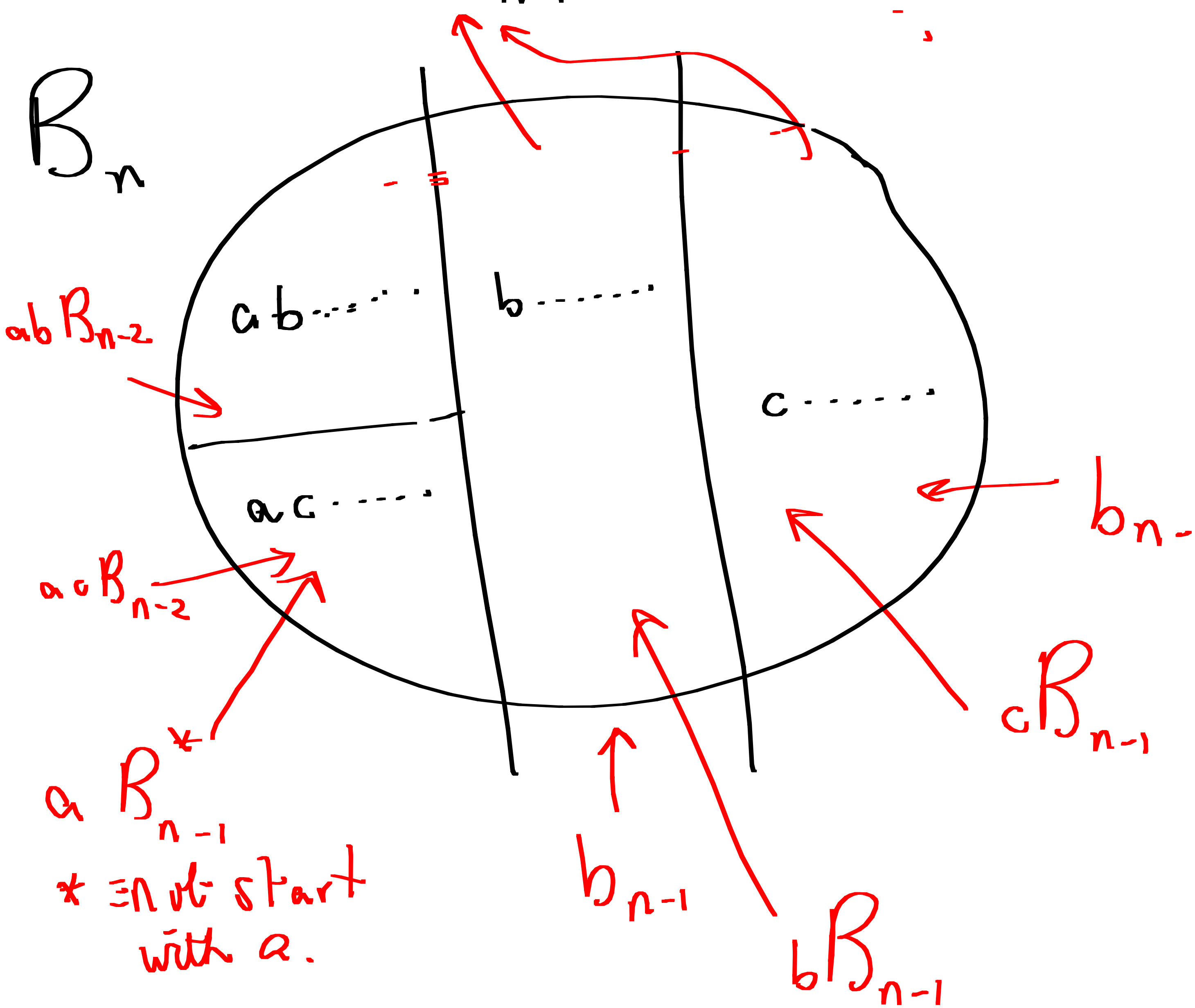
a
b
c

$b_2 = 8$

ab
ac
ba
bb
⋮
cc

$$b_n = 2b_{n-1} + 2b_{n-2}$$

$$b_n = 2b_{n-1} + 2b_{n-2}$$



A has \$n to spend.

Each day, buys Bun ($1), Ice-Cream ($2) or Pastry ($2).

Purchase: BBPIIPB

How many ways to spend money?

$$\underbrace{u_n = u_{n-1}}_{\text{Buy a Bun on day 1}} + \underbrace{2u_{n-2}}_{\text{Buy Pastry or Ice-Cream.}}$$

$u_0 = u_1 = 1$

Unknown sequence

$$a_0, a_1, a_2, \ldots, a_n, \ldots$$

equivalent to

Unknow function

$$a(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n + \cdots$$

Recurrence $a_0, a_1, \ldots a_n, \ldots$

leads to

Equation for $a(x)$

Unknown Sequence $\xrightarrow{\text{Recurrence}}$ Equation for gen. function

extract coefficients

Solve equation

$$(1+x)^\alpha = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n$$

True for arbitrary α

??. What is $\binom{\sqrt{2}}{4}$. $\dfrac{\sqrt{2}(\sqrt{2}-1)(\sqrt{2}-2)(\sqrt{2}-3)}{4!}$

$$u_n - 5u_{n-1} + 6u_{n-2} = 0 \qquad n \geq 2$$

$$u_0 = 1, \quad u_1 = 3 \qquad : \qquad u(x) = u_0 + u_1 x + \dots + u_n x^n + \dots$$

$$\sum_{n=2}^{\infty} \left(u_n - 5u_{n-1} + 6u_{n-2}\right) x^n = 0, \quad \forall x$$

$$\underbrace{\sum_{n=2}^{\infty} u_n x^n}_{u(x) - u_0 - u_1 x} - 5 \underbrace{\sum_{n=2}^{\infty} u_{n-1} x^n}_{x\left(u(x) - u_0\right)} + 6 \underbrace{\sum_{n=2}^{\infty} u_{n-2} x^n}_{x^2 u(x)} = 0$$

$$\sum_{n=2}^{\infty} u_{n-1} x^{n-1} = u_1 x + u_2 x^2 + \dots$$

$$\underbrace{\sum_{n=2}^{\infty} u_n x^n}_{u(x) - u_0 - u_1 x} - 5\underbrace{\sum_{n=2}^{\infty} u_{n-1} x^n}_{x\left(u(x) - u_0\right)} + 6\underbrace{\sum_{n=2}^{\infty} u_{n-2} x^n}_{x^2 u(x)} = 0$$

$$\sum_{n=2}^{\infty} u_{n-1} x^{n-1} = u_1 x + u_2 x^2 + \dots$$

$$u(x) - 1 - 3x - 5x\left(u(x) - 1\right) + 6x^2 u(x) = 0$$

$$u(x)\left[1 - 5x + 6x^2\right] = 1 + 3x - 5x$$
$$= 1 - 2x$$

$$u(x) = \frac{1-2x}{1-5x+6x^2}$$

$$U(x) = \frac{1-2x}{1-5x+6x^2}$$

$$= \frac{1-2x}{(1-2x)(1-3x)}$$

$$= \frac{1}{1-3x}$$

$$= \sum_{n=0}^{\infty} 3^n x^n$$

$$U_n = 3^n.$$