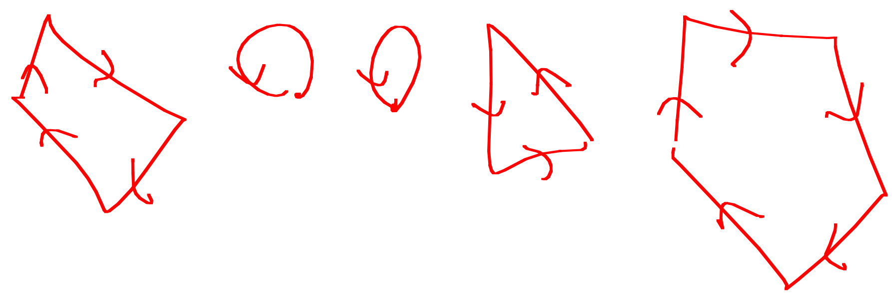
11\24\08

G

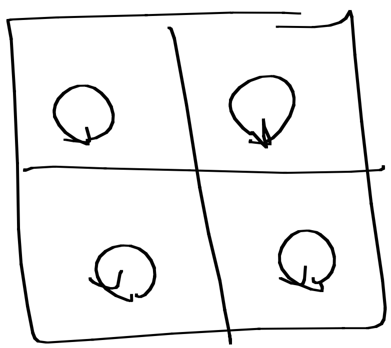
g is a permutation on D

g has cycles 

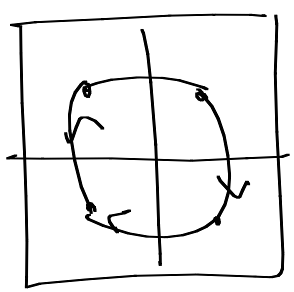
$$ct(g) = x_1^2 x_3 x_4 x_5$$

$$C_G = \sum_{g \in G} ct(g)$$

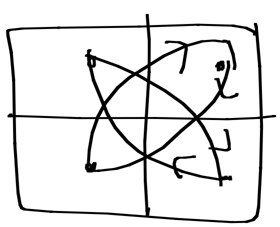
Example 2



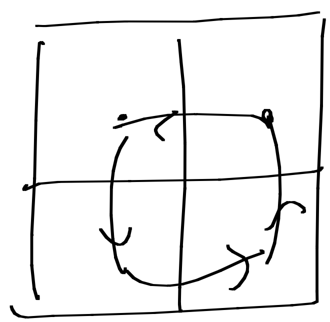
$e: \quad x_1^4$



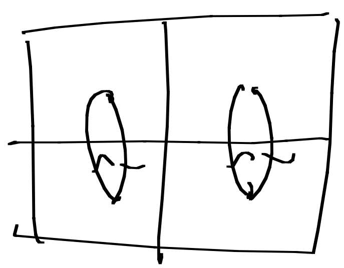
$a: \quad x_4$



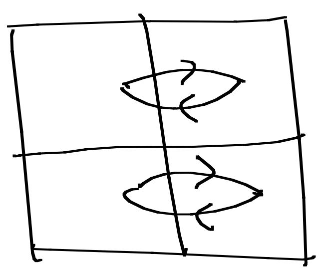
$b \quad x_2^2$



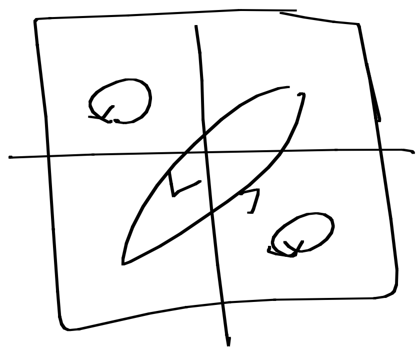
$c: \quad X_4$



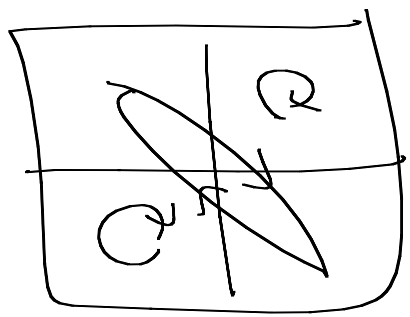
$p \quad X_2^2$



$q: \quad X_2^2$



$r : x_1^2 x_2$



$s : x_1^2 x_2$

$$C_G = \frac{1}{8}\left(x_1^4 + 3x_2^2 + 2x_1^2 x_2 + 2x_4\right)$$

$$C_G = \frac{1}{8}\left(x_1^4 + 3x_2^2 + 2x_1^2 x_2 + 2x_4\right)$$

Suppose we have color R, B, G

$x_1 \rightarrow R + B + G$

$x_2 \rightarrow R^2 + B^2 + G^2$

$x_3 \rightarrow R^3 + B^3 + G^3$

$x_4 \rightarrow R^4 + B^4 + G^4$

$$PI = \frac{1}{8}\left((R+B+G)^4 + 3(R^2+B^2+G^2)^2 + \cdots\right)$$

How colorings have 2R, 1B, 1G ?

= coefficient of $R^2 B G$

$$PI = \frac{1}{8}\left((R+B+G)^4 + 3(R^2+B^2+G^2)^2 + \cdots \right)$$

How colorings have 2R, 1B, 1G ?

$\equiv$ coefficient of R^2BG

$$(R+B+G)^4 = \frac{4!}{2!\,1!\,1!} = 12$$

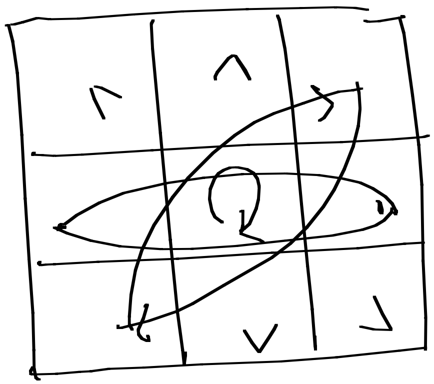
$$3(R^2+B^2+G^2)^2 = 0$$

$$2(R+B+G)^2(R^2+B^2+G^2) = 4$$

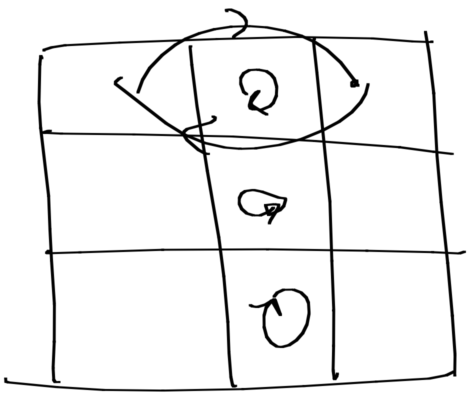
$$2(R^4+B^4+G^4) = 0$$

$$\# R^2BG = \frac{12+4}{8} = 2$$

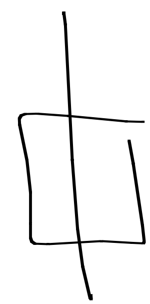
Example 2 $\quad n=3$



$b: \quad x_1 x_2^4$



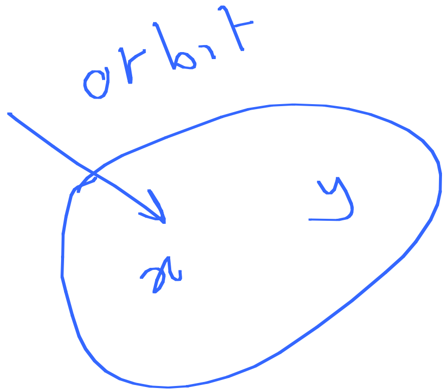
$x_1^3 x_2^3$



$$PI = \sum_{S^*} W(x)$$

S^* : choose one x from each orbit

coloring



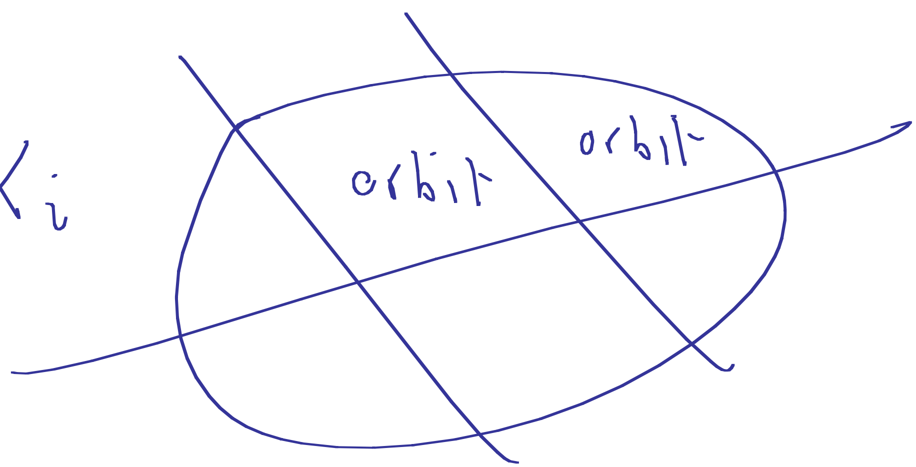
$W(x) = W(y)$

$$X = X_1 \cup X_2 \cup \cdots \cup X_m$$

$x, y \in$ same X_i,

iff $W(x) = W(y)$

X_i



$m_i = 6$

$$PI = \sum_{i=1}^{m} m_i W_i$$

m_i = # of orbits represented by X_i

W_i = common pattern of X_i

↓ Burnside

$$= \sum_{i=1}^{m} W_i \left[\frac{1}{|G|} \sum_{g \in G} \left| Fix\left(g^{(i)}\right) \right| \right]$$

g acting on X_i

$$= \frac{1}{|G|} \sum_{g \in G} \underbrace{\left(\sum_{i=1}^{m} \left| Fix\left(g^{(i)}\right) \right| \right) W_i}_{W(Fix(g))}$$

g: $\circlearrowleft C_3$ $\circlearrowleft C_4$

$(R^4 + B^4 + \dots)$ $(R^3 + B^3 \dots)$