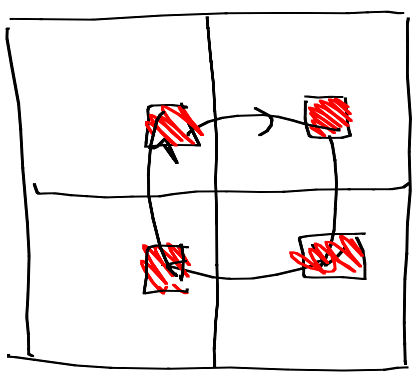
11\21\08

$$\nu_{X,G} = \frac{1}{|G|} \sum_{g \in G} |Fix(g)|$$

$n = 2m$



2 colors

g	$\lvert Fix(g) \rvert$
e	2^{n^2}
a 90°	$2^{n^2/4}$
b	
c	
p	
q	
r	
s	

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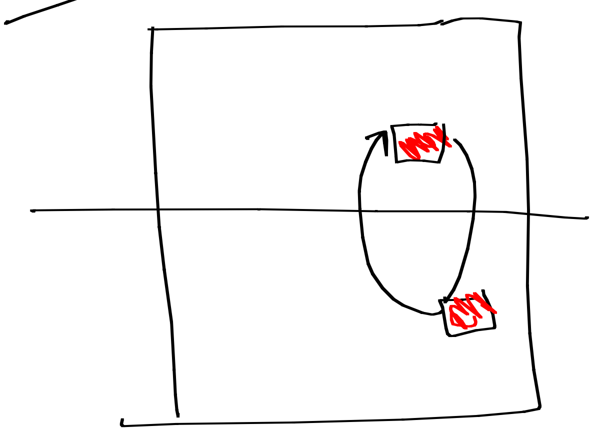
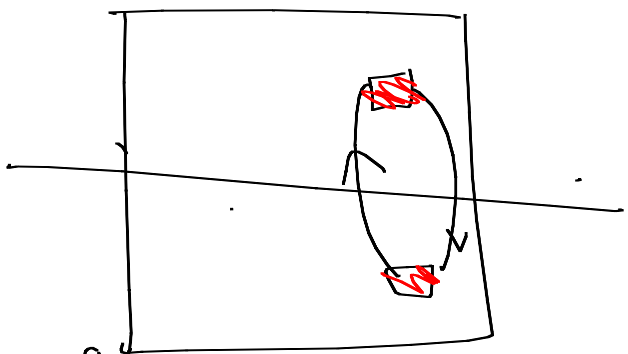
b

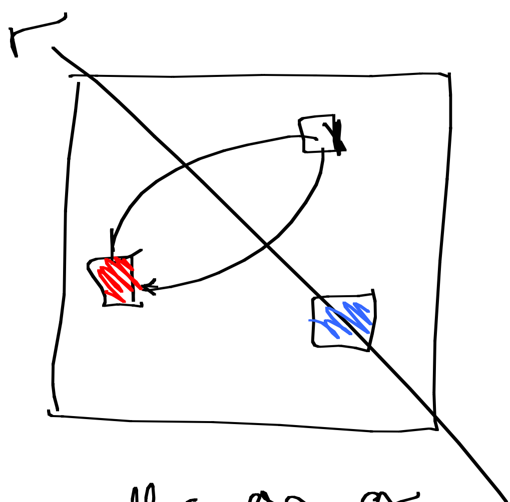


c ✓

P & Q



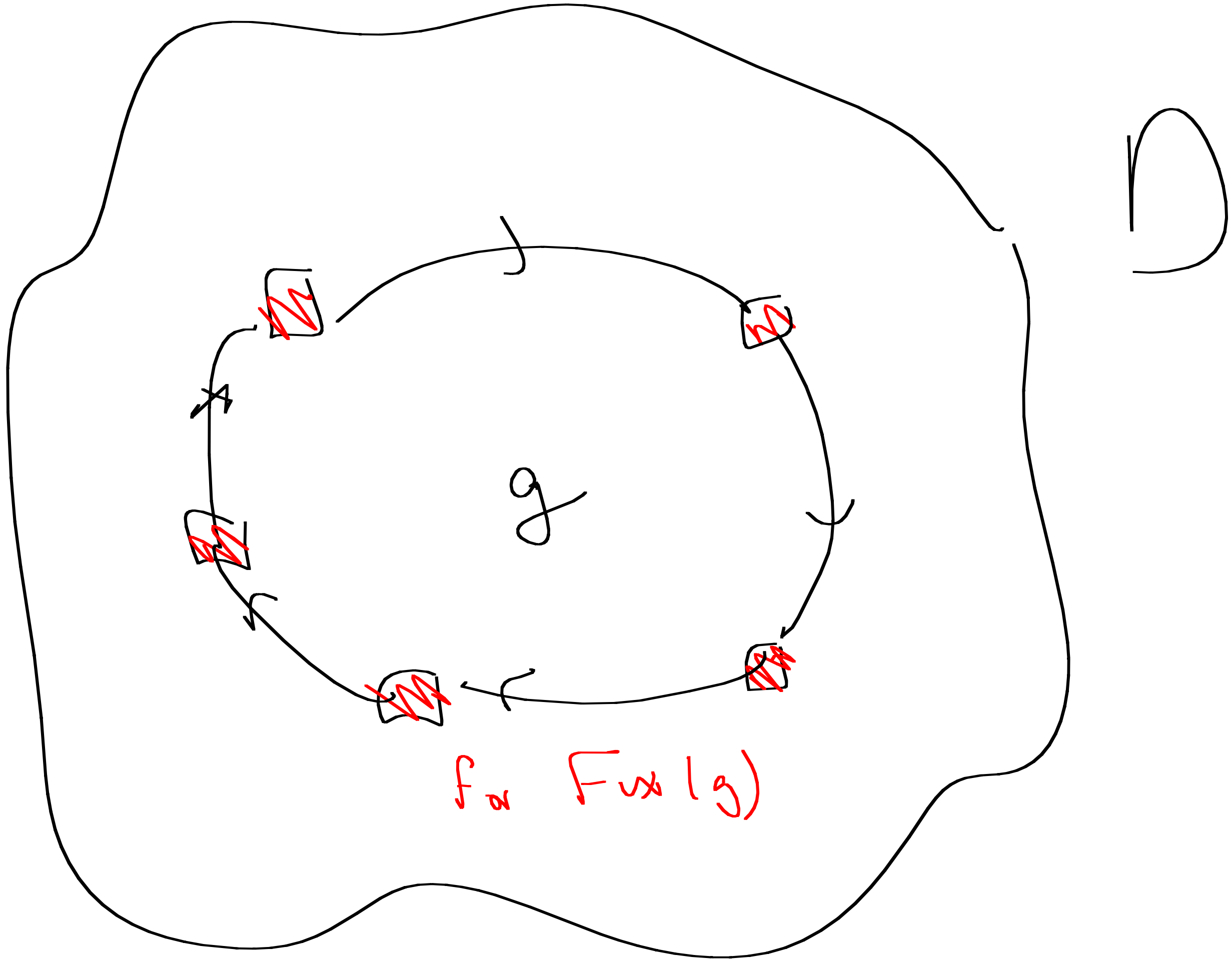
$n = 2l$



cells on or below diagonal is $\frac{n(n+1)}{2}$

k colors

g	$\lvert \mathrm{Fix}(g) \rvert$	
e	2^{n^2}	k^{n^2}
90° a	$2^{n^2/4}$	$k^{n^2/4}$
b	$2^{n^2/2}$	⋮
c	$2^{n^2/4}$	⋮
p	$2^{n^2/2}$	⋮
q	$2^{n/2}$	⋮
r	$2^{n(n+1)/2}$	
s	$2^{n/(n+1)/2}$	



g is partitioning D into cycles.

All things in same cycle have same color

Cycles of a permutation

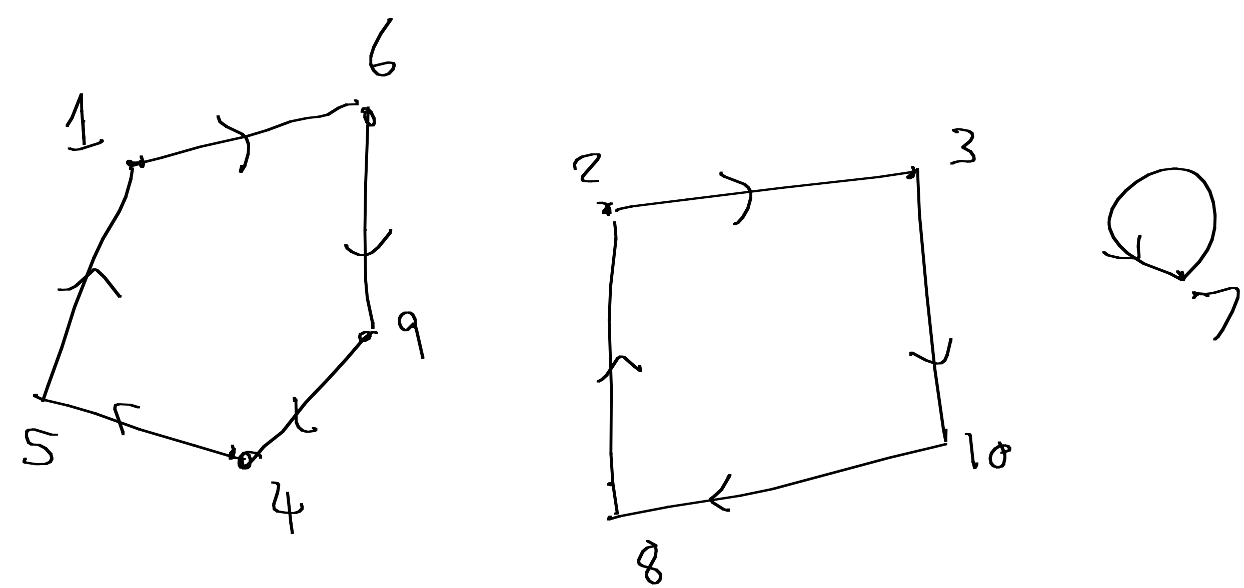
$\pi : D \longrightarrow D$

Γ_π is a digraph $\quad i \longrightarrow \pi(i)$

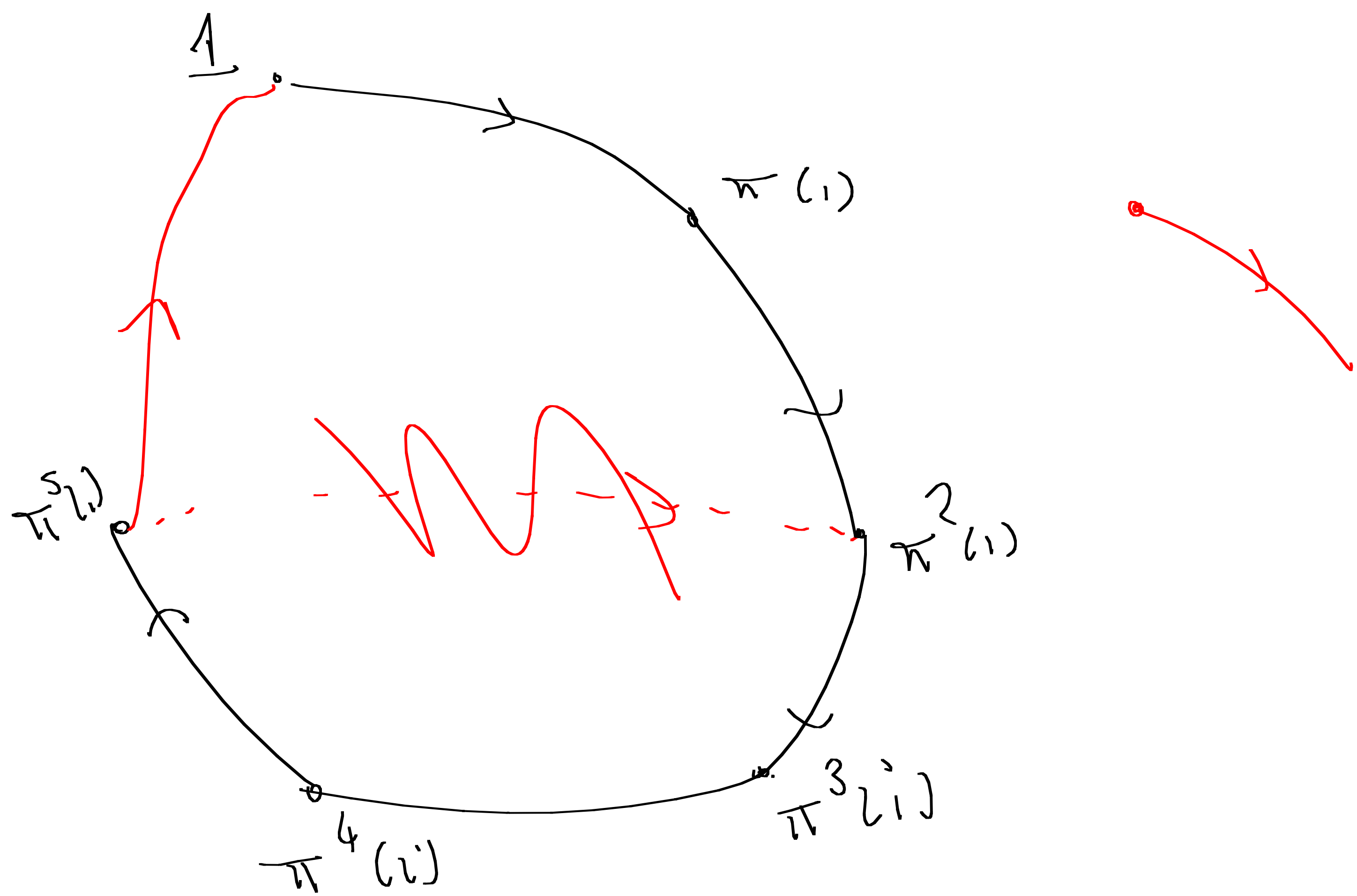
$\boxed{\text{Union of disjoint cycles}}$

Example

i	1	2	3	4	5	6	7	8	9	10
$\pi(i)$	6	3	10	5	1	9	7	2	4	8



Γ_π



Polya's Theorem

D omain $\qquad X = \{x : D \to C\}$

Colors $\qquad$ set of coloring

Group $G = \{$permutations of $D\}$

"Induces" permutation on X

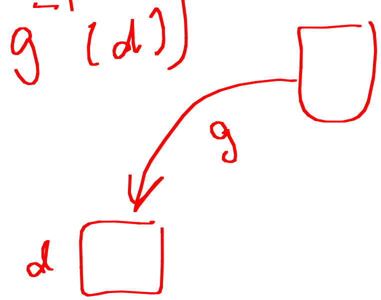
$x \in X$

$g * x = ?$

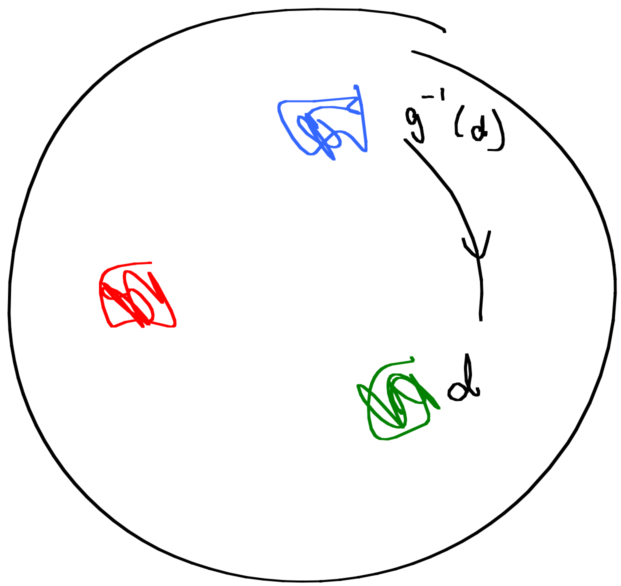
$x \in X$
$g * x \in X$

$g * x(d) = x(g^{-1}(d))$

color of d
under $g * x$



D + coloring = x



$g : D \longrightarrow D$

New coloring

$g * x$

$g * x(d) = x(g^{-1}(d))$

Goal

We know how to count "distinct" colorings of X

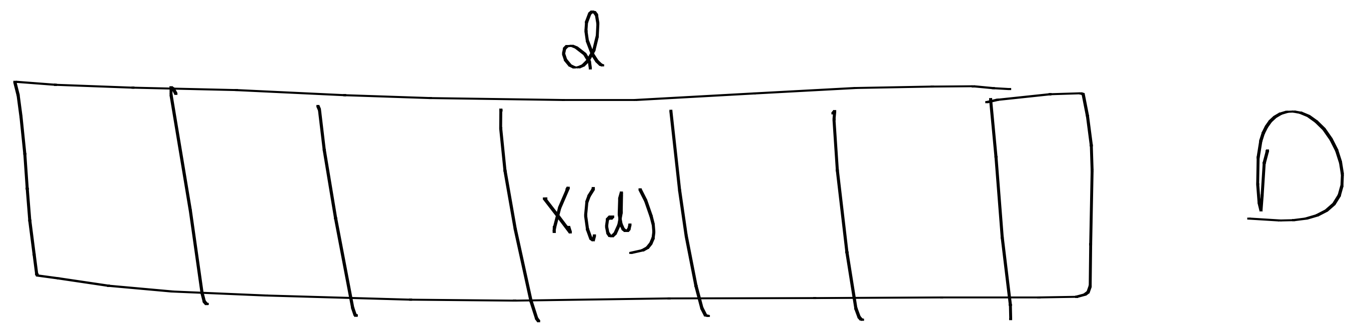
Suppose we have 2 color, R & B

$|D| = 100$

How many "distinct" colorings have $60\,R$ & $40\,B$.

Pattern Inventory

$x \in X$ colors are $\{R, B, G, \ldots$

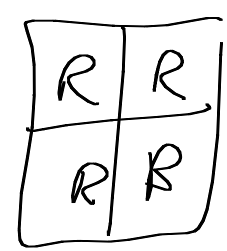


$\omega_{x(d)} \equiv$ "weight" of a color.

If $C = \{R, B, G, \ldots\}$

$\omega_R = R$, $\omega_B = B$, ...

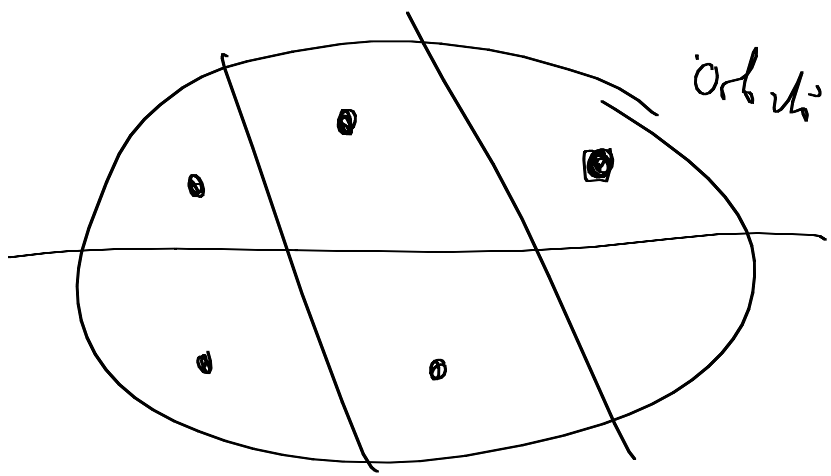
$\omega(x) = \prod_d \omega_{x(d)}$



$R^3 B$

$$PI = \sum_{x \in S^*} \omega(x)$$

S^* contains one coloring from each orbit.



Coefficient of R^3B^3 = # of distinct colorings with 3R & 3B

Ex: 2

$$PI = R^4 + R^3B + 2R^2B^2 + RB^3 + B^4$$