11/12/08

A game:

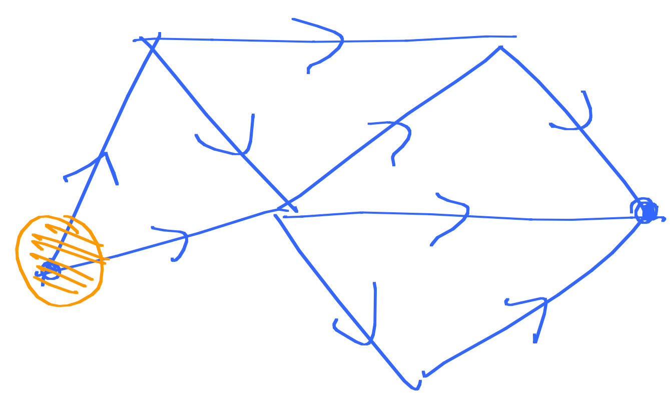
$$D = (X, A)$$

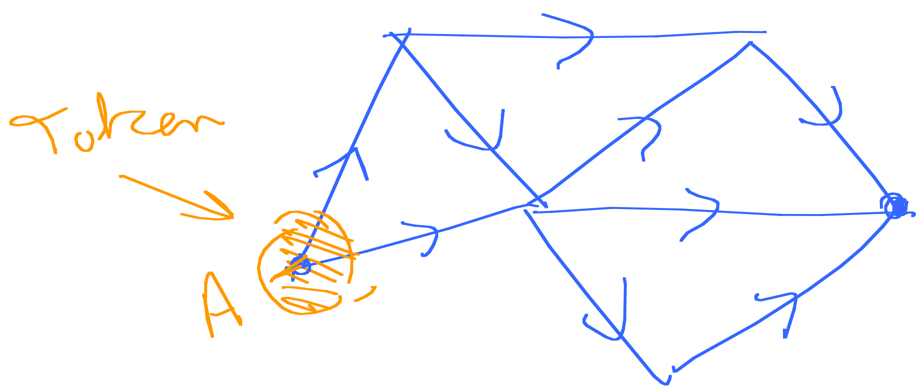
positions of game

moves: $(x, y) \in A$

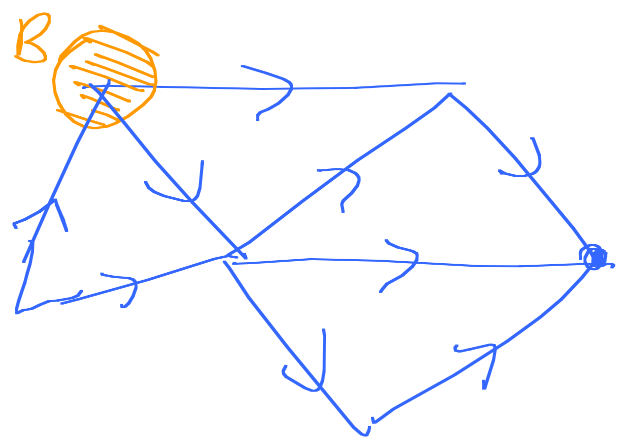
can move from $x \to y$

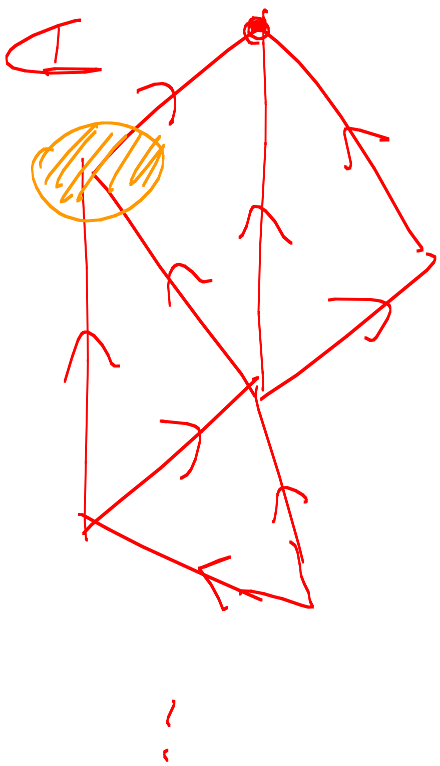
Token

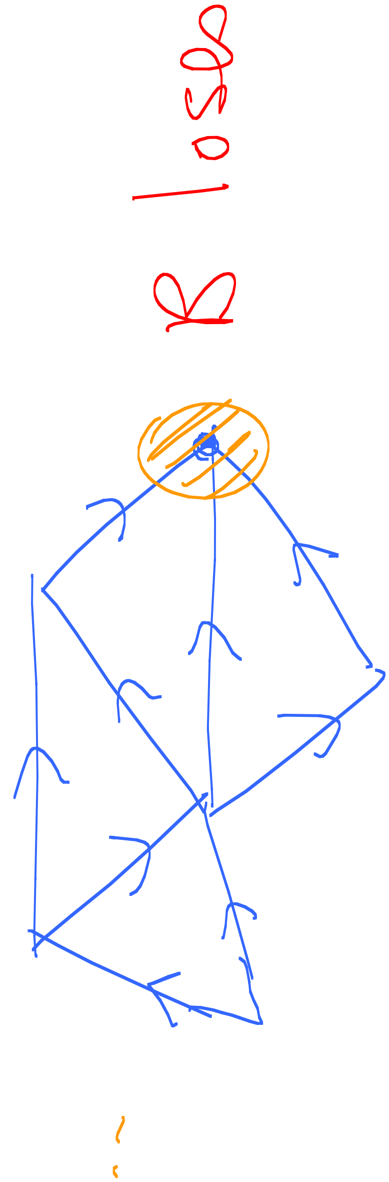




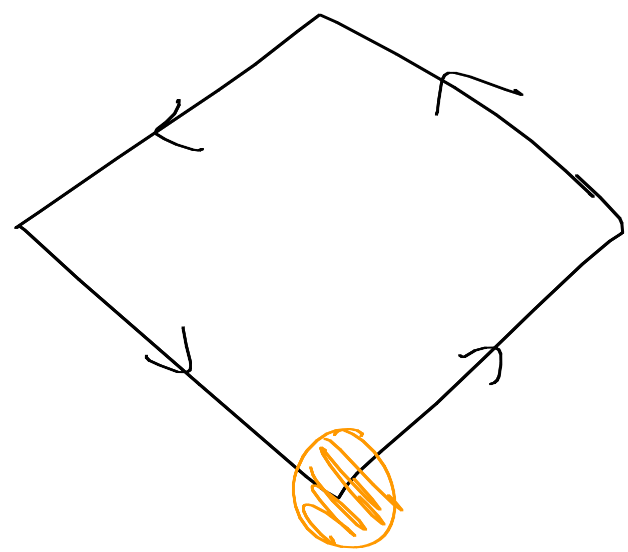
Each player, takes turns in moving token: Winner = last person to move




I


R loss

Does a game end?



This game won't end

Assume $|X| < \infty$

and no directed cycle: DAG.

Topological numbering

$|X| = n$

label X with $1, 2, \cdots, n$

such that

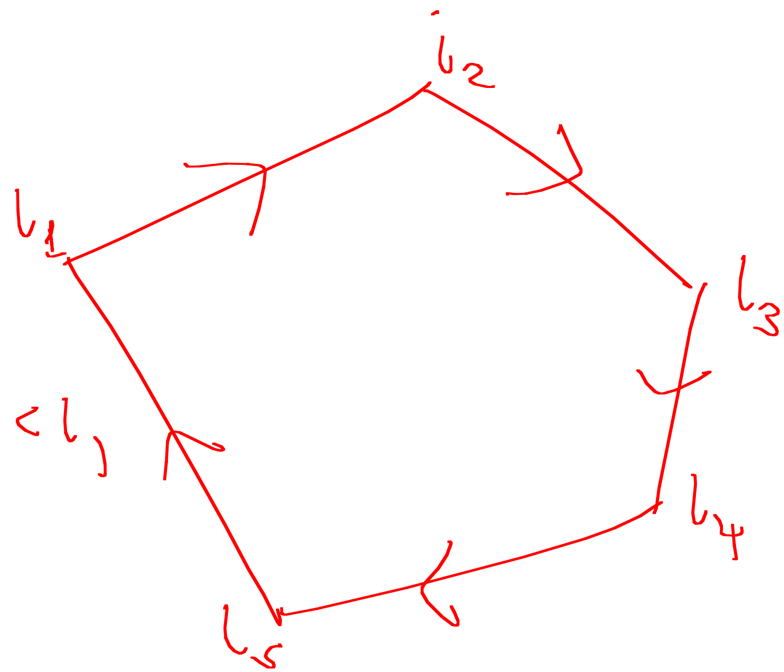
$$i \longrightarrow j$$

$$\Rightarrow i < j$$

A digraph has no cycles [is a DAG] iff $\exists$ a topological numbering.

(1) Suppose $\exists$ top. num.

No cycles:



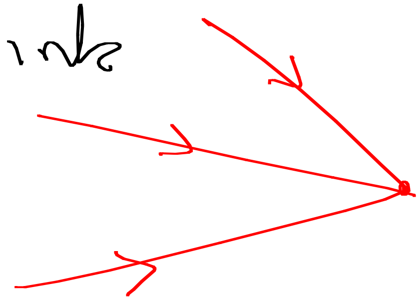
$l_1 < l_2 < l_3 < l_4 < l_5 < l_1$

??

No cycle $\Longrightarrow$ $\exists$ top. num.

$\Downarrow$

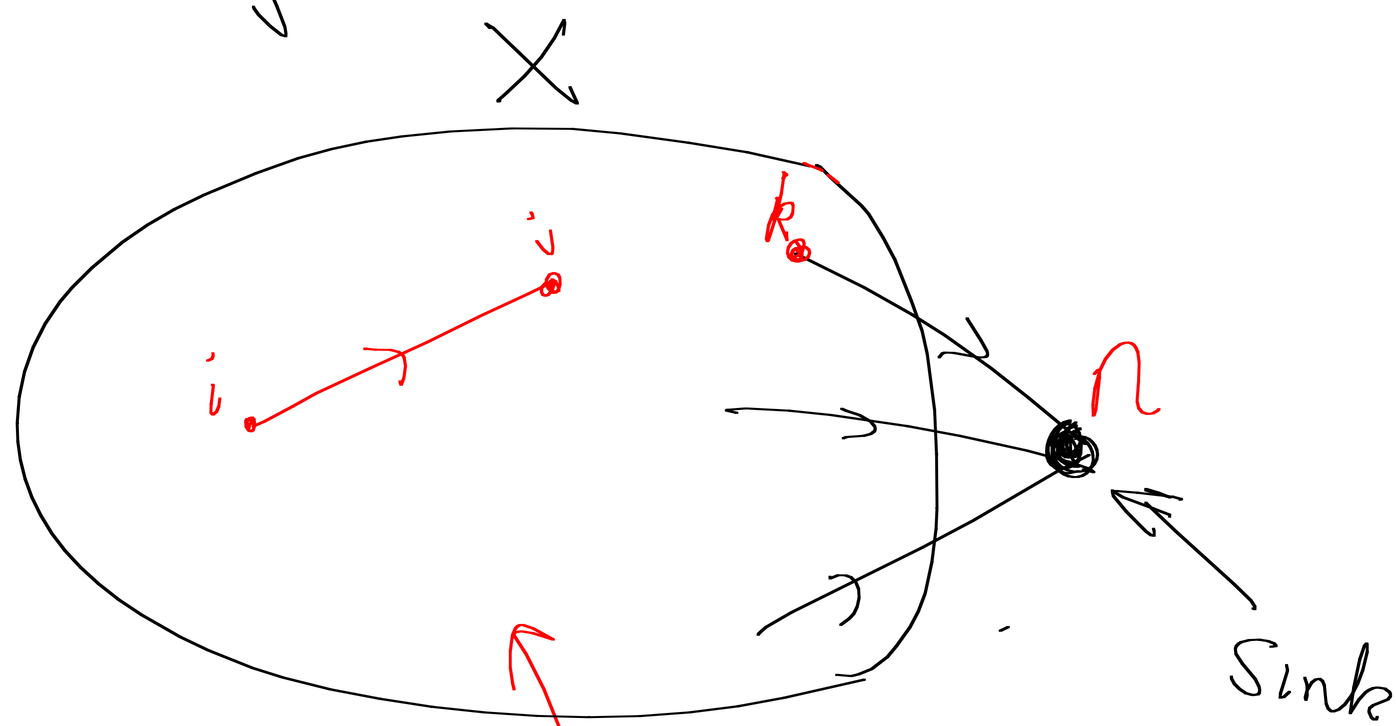
$\exists$ Sink



Outdegree = 0

P = longest path

We can now use induction to get numbering.



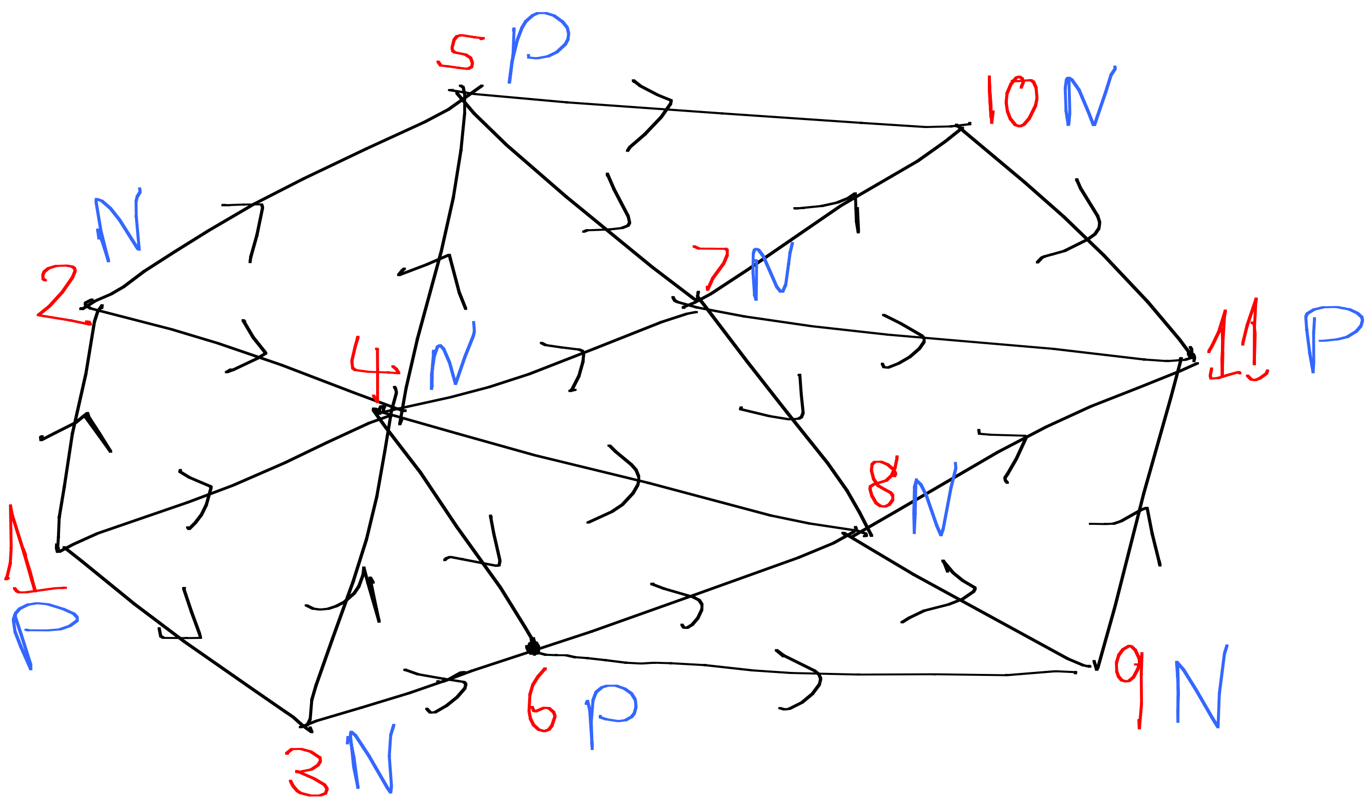
$\exists$ numbering of , by induction on $|X|$.

$x \in N$ iff $N^+(x) \cap P \neq \emptyset$

Given game (X, A)

we want to compute partition

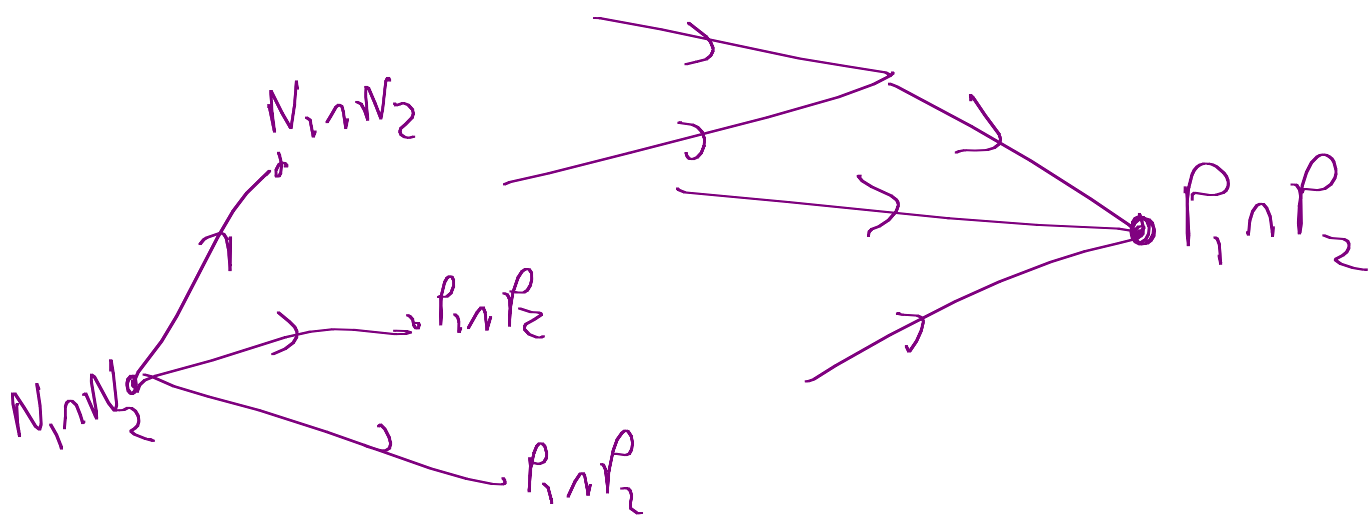
$$X = N \cup P$$



$$x \in N \iff N^{+}(x) \cap P \neq \emptyset$$

N, P satisfying is unique.

Suppose $\exists\ (N_1, P_1)\ \&\ (N_2, P_2)$

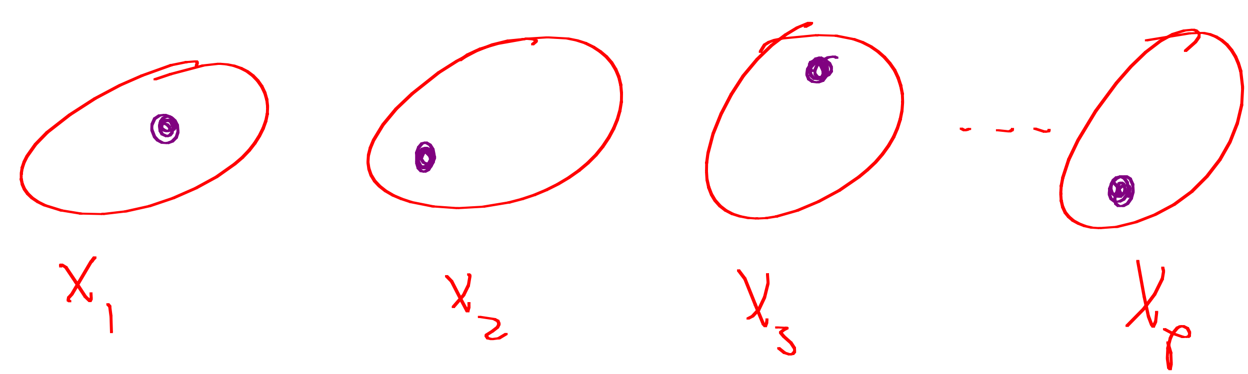


Sum of games

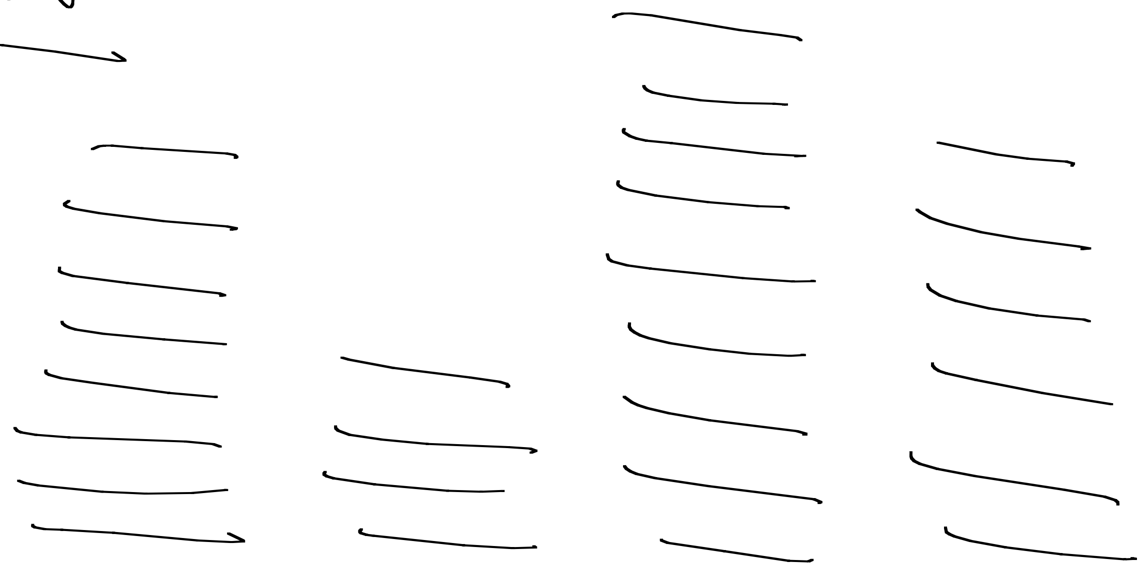
$(X_1, A_1) \quad (X_2, A_2) \quad \cdots \quad (X_p, A_p)$

$G_1 \qquad G_2 \qquad \qquad G_p$

$$G_1 \oplus G_2 \oplus \cdots \oplus G_p$$



Nim



Move : take coins from non-empty pile.

Single game easy to play: Take whole pile

Knowing N_i, P_i does not yield solution easily.