11\10\2008

Schur's Theorem

$$r_k = N(\underbrace{3, 3, \ldots, 3}_{k}; 2)$$

$n \geq r_k$ & you k- color edges of K_n, then $\exists$ mon- triangle

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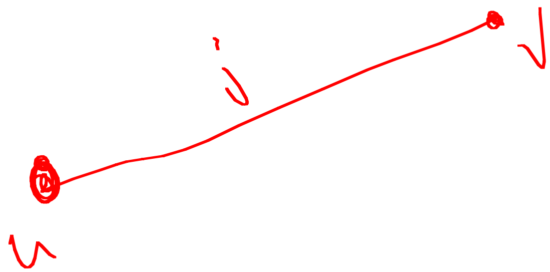
Suppose $n \geq r_k$ and $[n] = S_1 \cup S_2 \cup \ldots \cup S_k$

$\Rightarrow \exists i$ and $x, y, z \in S_i$ s.t.

$$x + y = z$$

mono chromatic solution

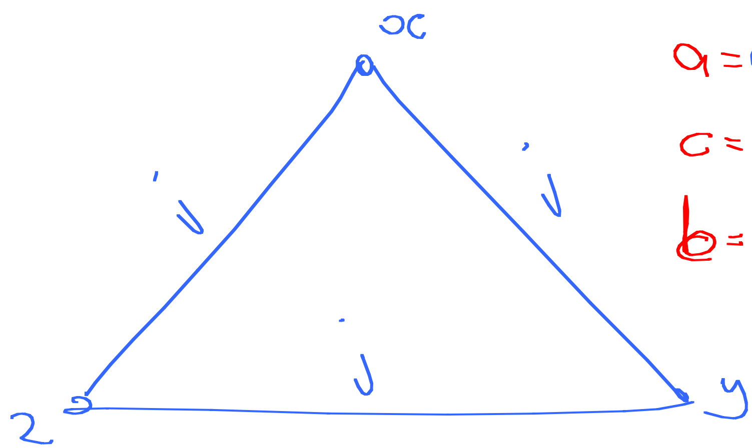
Color edges of K_n



$|u-v| \in S_j$

$\Rightarrow \exists$ mon-triangle

Assume $x < y < z$



$a = y - x \in S_j$

$c = z - x \in S_j$

$b = z - y \in S_j$

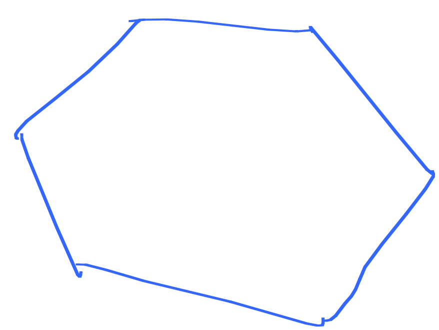
$a + b = c$

Geometric Question

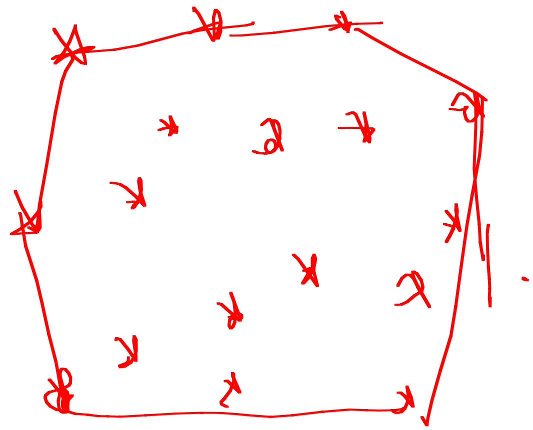
n points $X_1, X_2, \ldots, X_n$ in plane. [No 3 are in a line]

$n \geq N(k,k;3)$ — 2 coloring triples

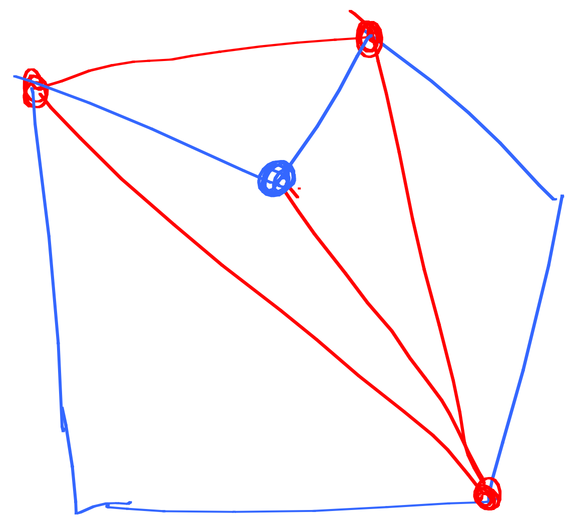
$\Rightarrow \exists$ k points forming a convex polygon



$\leftarrow$ k vertices.



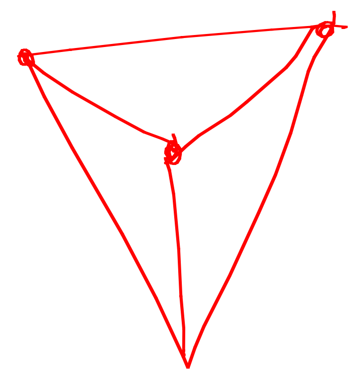
Observation:



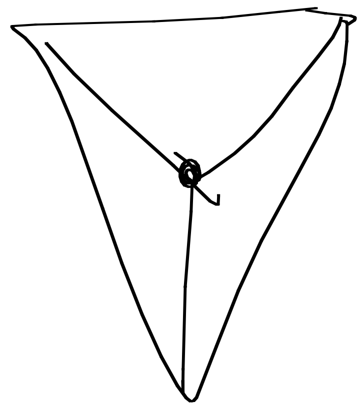
Non-convex

$\Downarrow$

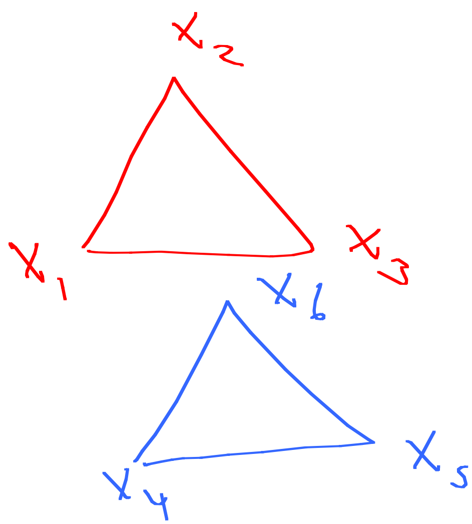
$\exists$ 4 pts

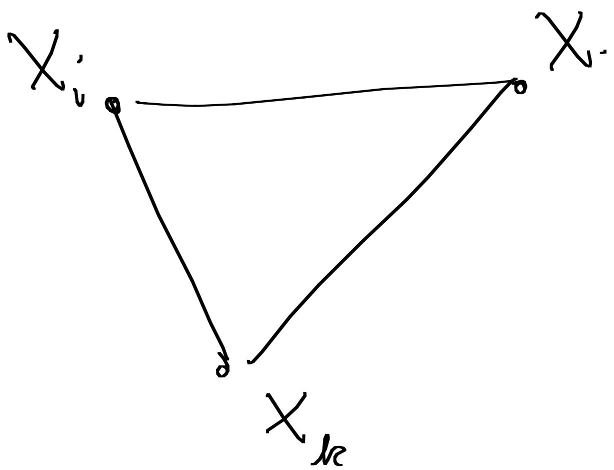


We show $\exists$ k pts without



$n \geq R(k,k;3)$





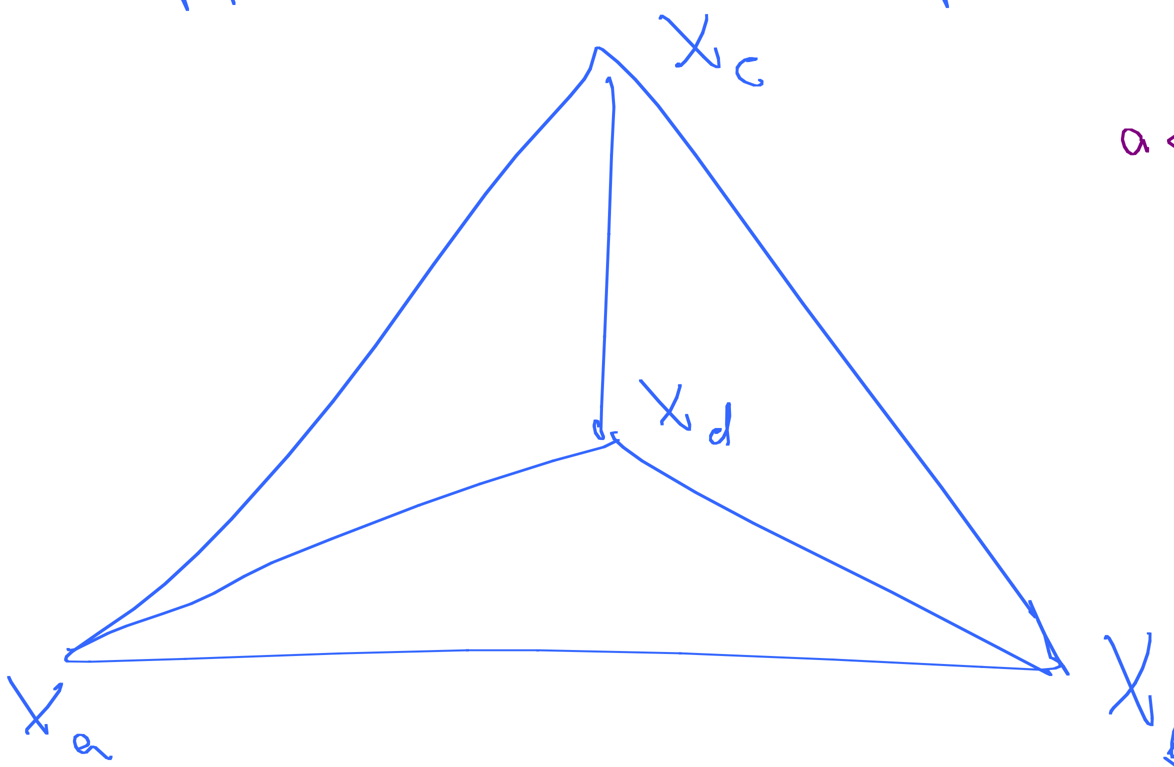
Traverse Δ

$X_i \to X_j \to X_k$

There must be a mono k set.

Suppose there are points X_a, X_b, X_c, X_d.

$a < d < c$



Assume: $a < b < c$ and all Δ's are blue

<u>Combinatorial Games</u>

<u>Game 1</u>

n chips, 2 players A, B

Take turns to remove some chips.

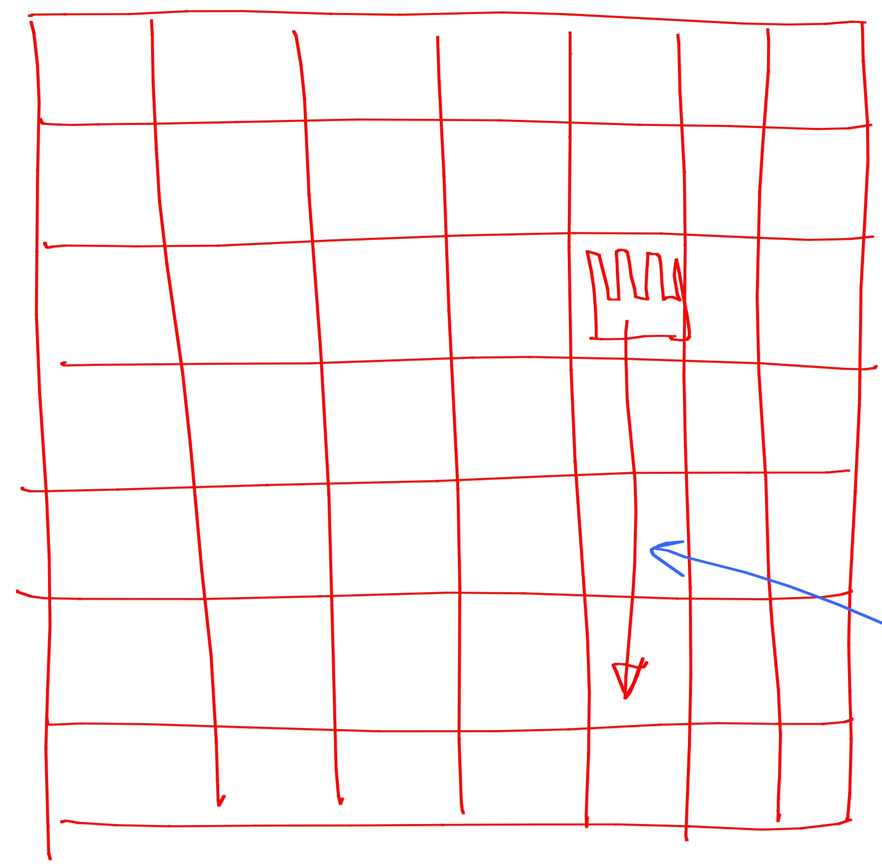
chips to take $= 1, 2, 3, 4$.

Person who takes last chip <u>wins</u>

Lose if you cannot move.

Winning Positions	Losing Positions
$n = \underline{1}, 2, 3, 4$	5
$6, 7, 8, 9$	10

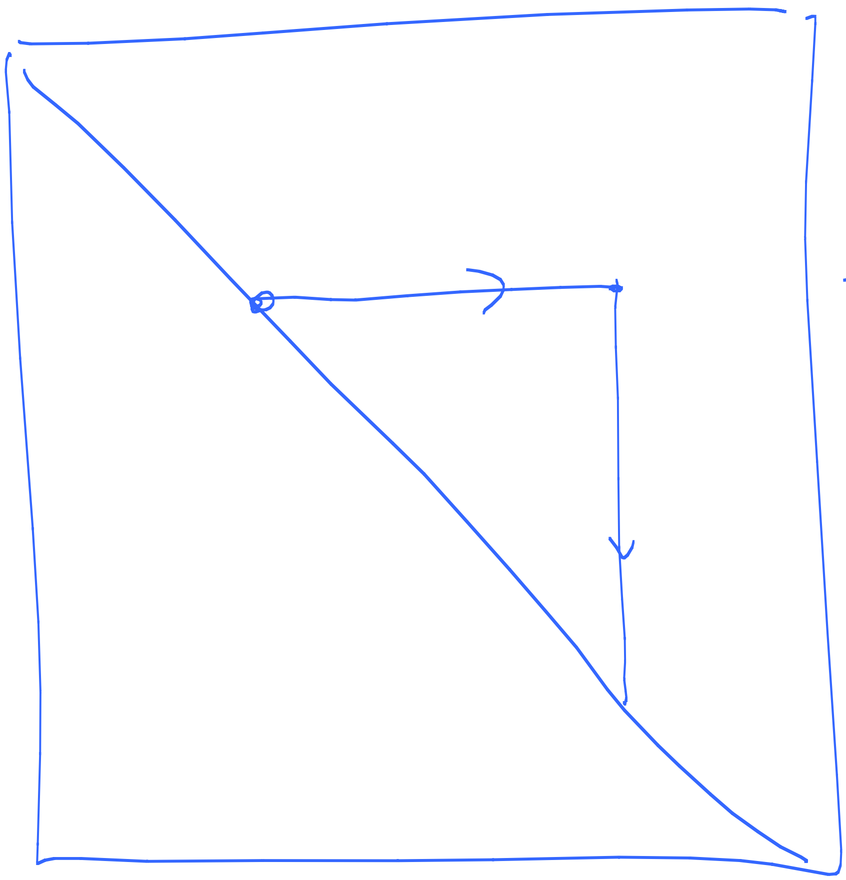
$\vdots$

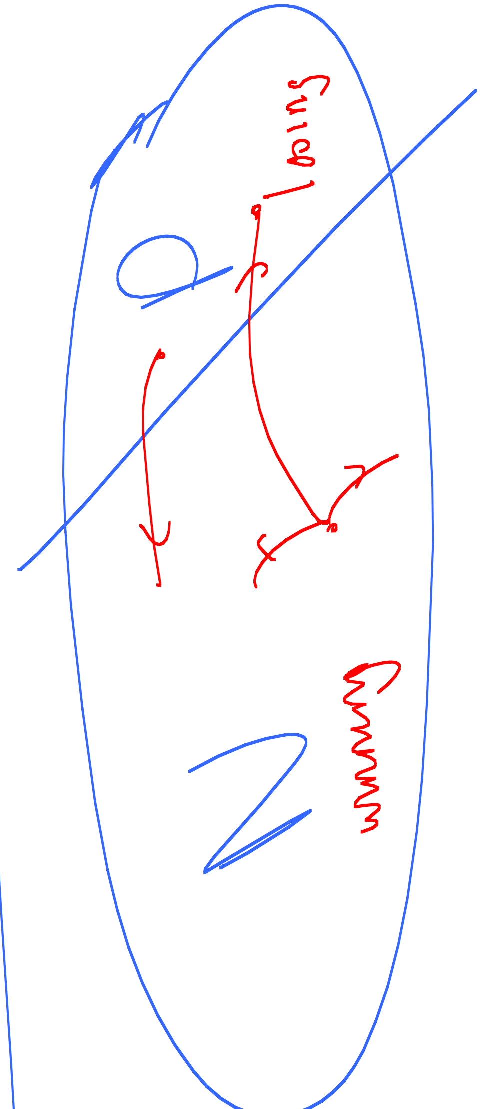


remove 3
from pile
2.

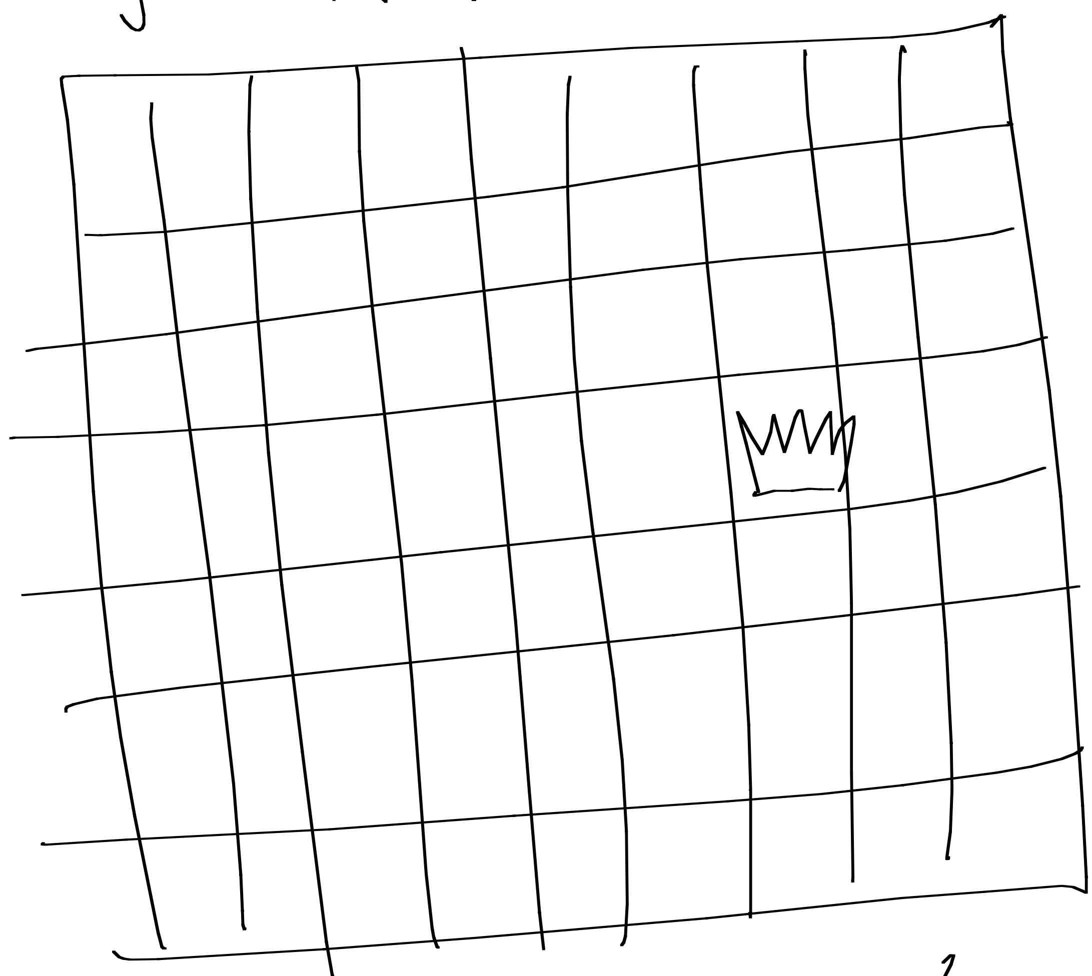
Two piles with m chips
n chips

Take from one of the piles





Wythoff's Nim



$$(m, n) \longrightarrow \begin{array}{ll} (m', n) & m' < m \\ (m, n') & n' < n \\ (m-a, n-a) & a > 0 \end{array}$$

Geography

$W = \{\text{words}\}$

names of countries

Players: A Albania afghanistan luxemburg

B armenia nepal