10\24\08

$$[n]^{n-2} \longrightarrow \text{Trees with vertex set } [n]$$

$$s_1 s_2 \cdots s_{n-2} \longrightarrow T$$

v appear $d_T(v) - 1$ times

trees T with $d_T(i) = d_i$, $\forall i$

= # sequence in which i appears $d_i - 1$ times, $\forall i$

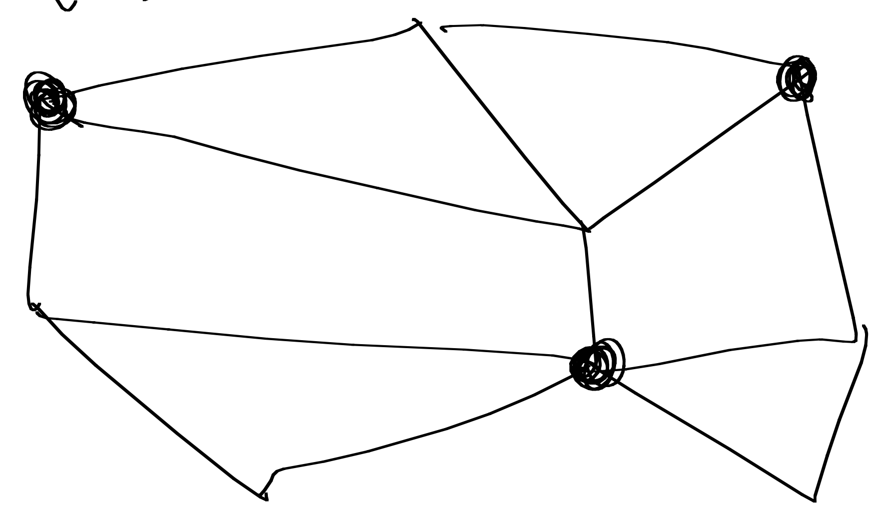
$$= \binom{n-2}{d_1 - 1, \ldots, d_n - 1} \qquad n-2 = (d_1 - 1) + (d_2 - 1) + \cdots + (d_n - 1)$$

Turan's Theorem

$S \subseteq V$ is independent in graph G

if S contains no edges.

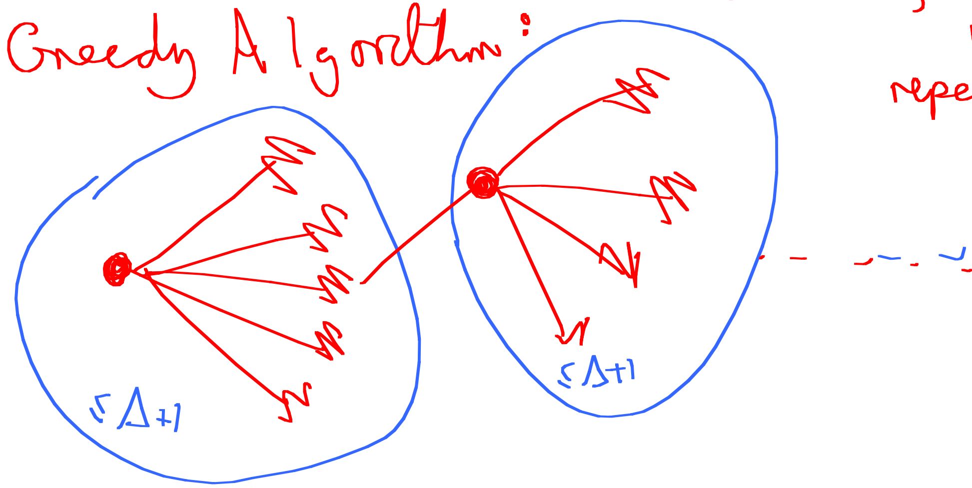
$\alpha(G)$ = size of largest independent set.



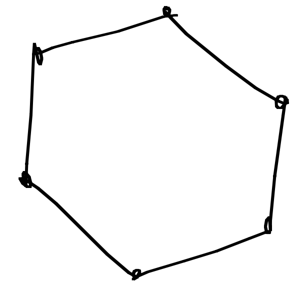
Suppose maximum degree in G is Δ.

$$\alpha(G) \geq \frac{n}{\Delta+1} \qquad n = |V|$$

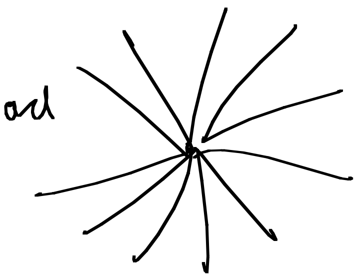
Greedy Algorithm: choose vertex, remove nbrs; repeat.



$\frac{n}{\Delta+1}$ is good if Δ is small



bad



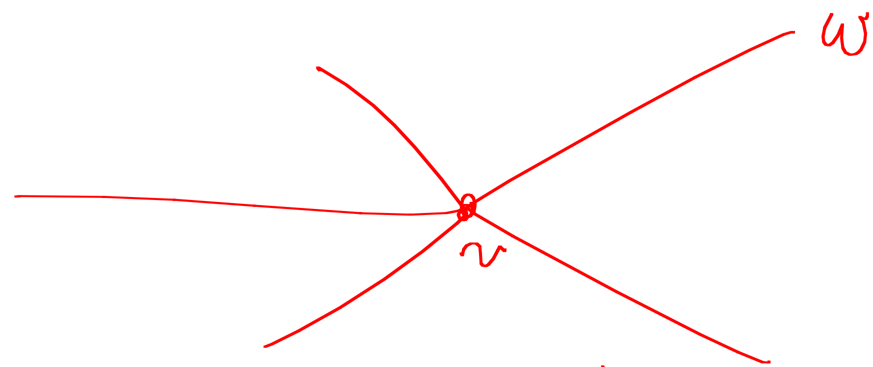
__Turan__

$$\alpha(G) \geq \frac{n}{\text{average degree} + 1}$$

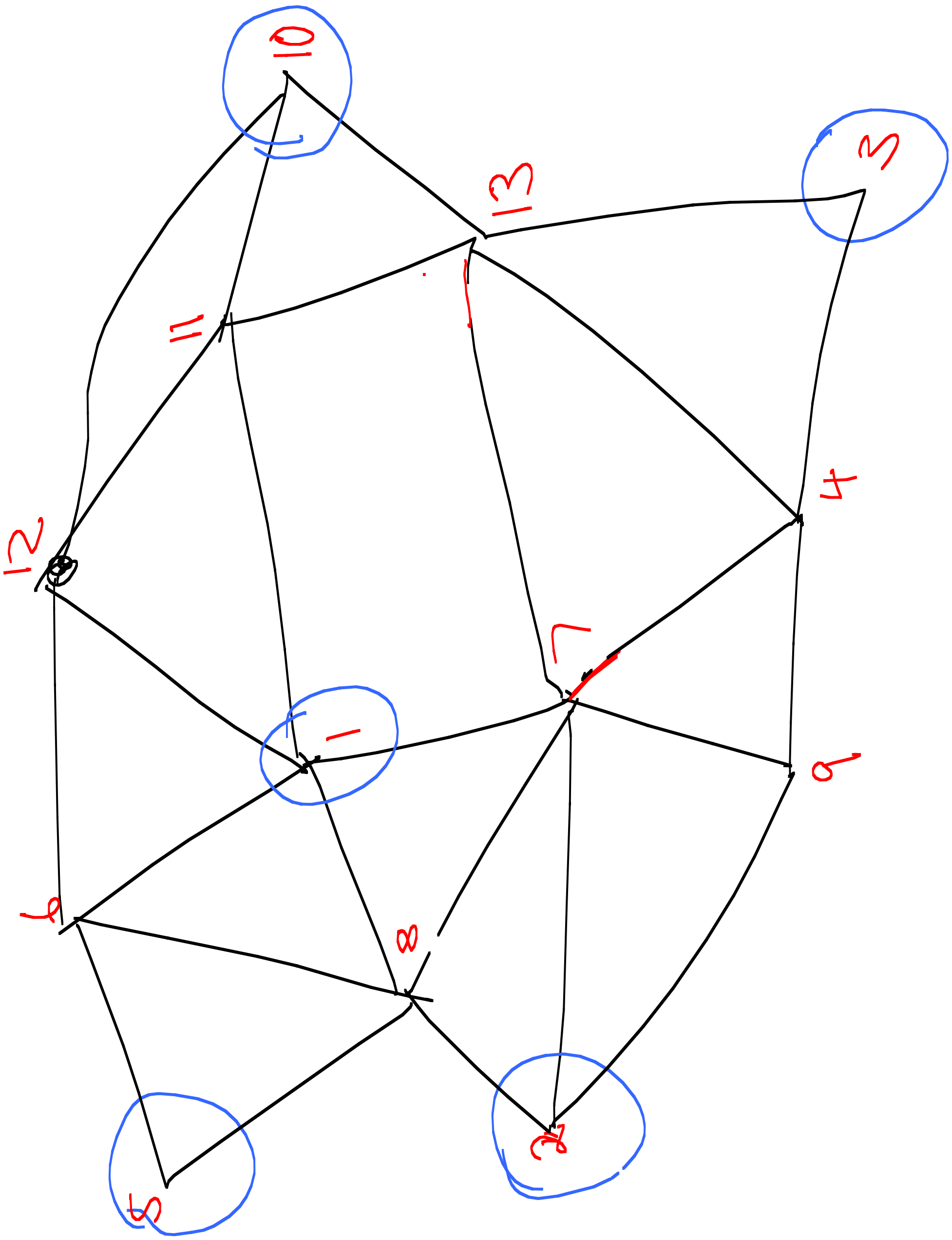
m edges $\frac{n}{\frac{2m}{n}+1}$

If Π is a permutation of vertices $1, 2, \ldots, n$ we can get an independent set:

$$I(\pi) = \{ v : \pi(w) > \pi(v), \forall w \in N(v) \}$$



$v \in I(\pi)$ if $\pi(v) < \pi(w), \forall w$



10
13
3
11
12
4
7
1
9
6
8
2
5

Choose π at random.

Compute

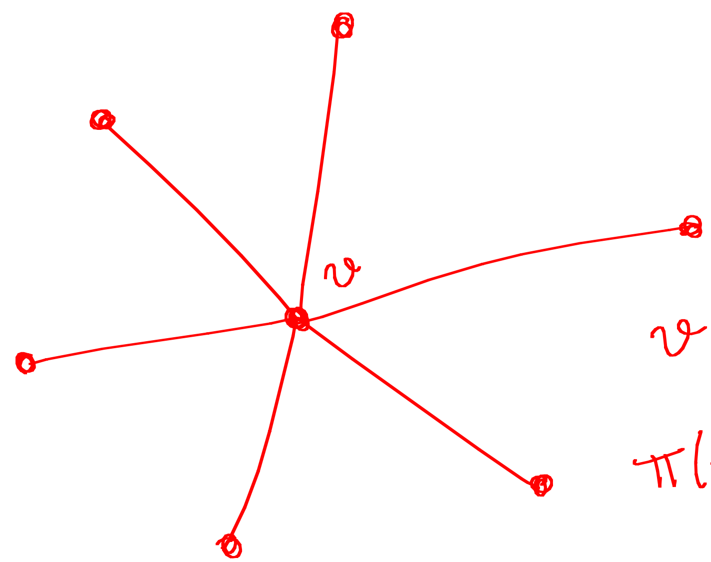
$$E\left(\left|I(\pi)\right|\right) = E\left(\sum_{v \in V} \mathbb{1}_{v \in I(\pi)}\right)$$

$$= \sum_{v \in V} E\left(\mathbb{1}_{v \in I(\pi)}\right)$$

$$= \sum_{v \in V} \Pr\left(v \in I(\pi)\right)$$

$$E\left(\mathbb{1}_{v \in I(\pi)}\right) = 1 \times \Pr\left(v \in I(\pi)\right) + 0 \times \Pr\left(v \notin I(\pi)\right)$$

$$\Pr\big(v \in I(\pi)\big) = \frac{1}{d_v + 1}$$



$v \in I(\pi)$ iff

$\pi(v) = \min\{\pi(x) : x \in v + N(v)\}$

$$E(I[\pi]) = \sum_{v \in V} \frac{1}{d_v + 1}$$

$$\Rightarrow$$

$$\alpha(G) \geq \sum_{v \in V} \frac{1}{d_v + 1}$$

$$\overset{?}{\geq} \frac{n^2}{2m+n} \quad ??$$

To prove

$$\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_k} \geq \frac{k^2}{x_1 + \cdots + x_k}$$

Harmonic Mean $\leq$ Arithmetic Mean

$$\frac{1}{\left(\frac{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_k}}{k}\right)} \leq \frac{x_1 + x_2 + \cdots + x_k}{k}$$

Apply

$$x_v = \frac{1}{d_v + 1} \; ; \; \sum_v \frac{1}{d_v + 1} \geq \frac{n^2}{(d_1 + 1) + \cdots + (d_n + 1)}$$

$$= \frac{n^2}{2m + n}$$

To prove

$$\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_k} \geq \frac{k^2}{x_1 + \cdots + x_k}$$

Multiply through by $x_1 + x_2 + \cdots + x_k$

$$\Uparrow$$

$$\sum_{i \neq j} \frac{x_i}{x_j} + \underbrace{\sum_{i=j} \frac{x_i}{x_i}}_{k} \geq k^2$$

$$\Uparrow$$

$$\sum_{1 \leq i < j \leq k} \underbrace{\left(\frac{x_i}{x_j} + \frac{x_j}{x_i} \right)}_{\geq 2} \geq k(k-1)$$

$$\boxed{y + \frac{1}{y} \geq 2 \qquad y > 0}$$

If $m > \dfrac{(k-2)n^2}{2(k-1)}$ then G has a clique of size k.

$\bar{G}$ = complement of G.

Suppose G has no clique of size k.

$$k-1 \geq \alpha(\bar{G}) \geq \frac{n^2}{2\left[\binom{n}{2} - m\right] + n}$$