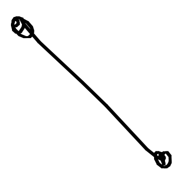
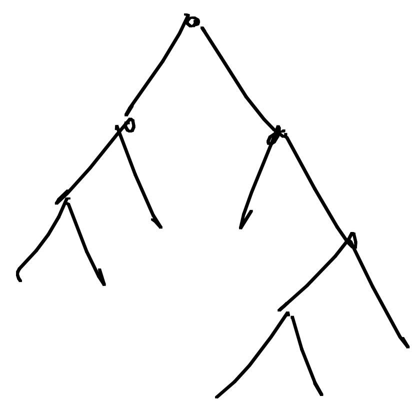
10\22\08

A tree is a connected graph without cycles.

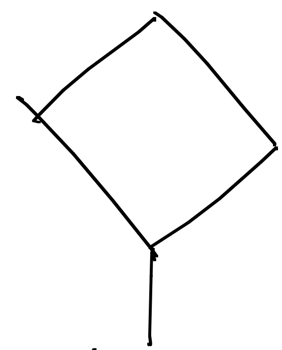
Tree



Tree



Tree



NOT A TREE

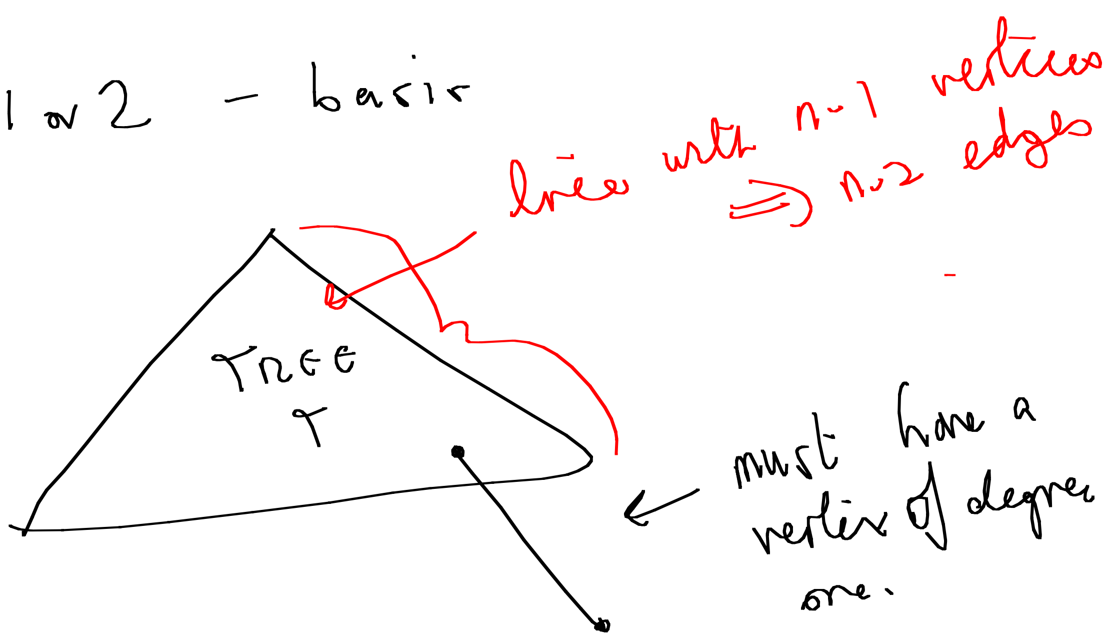
Properties of a tree T

- If T has n vertices then it has $n-1$ edges.

"Proof"

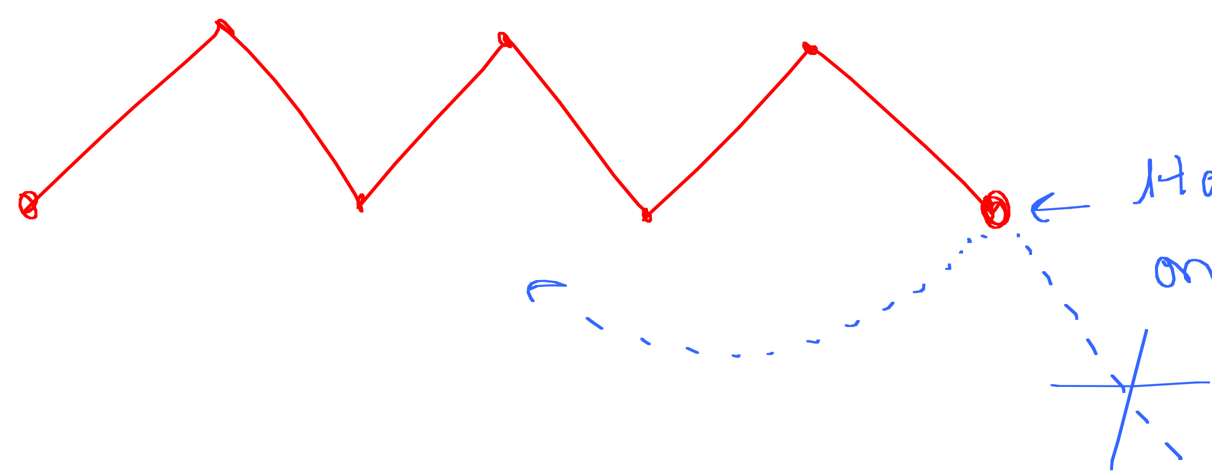
True if $n = 1$ or 2 — basis

Induction



must have a vertex of degree one.

Let P be a longest path in T

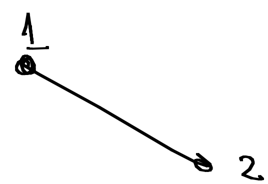


Has degree one in T.

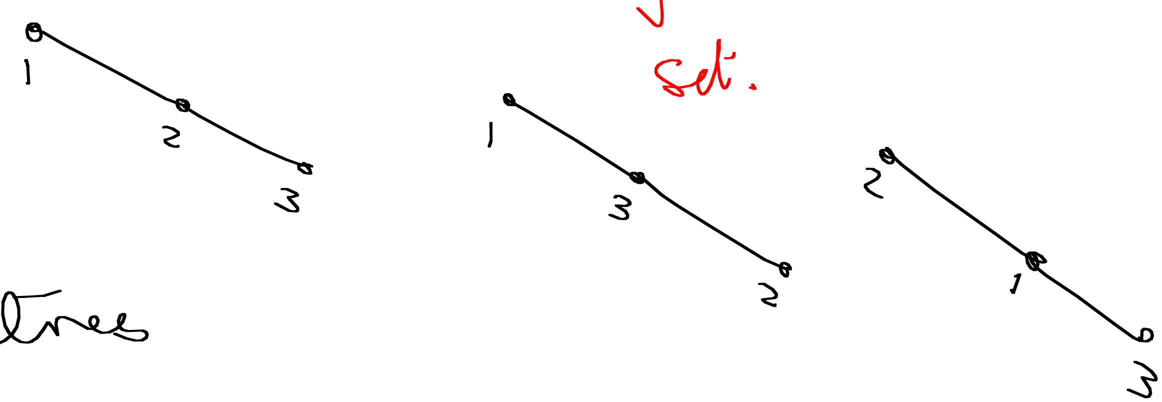
other P is not longest

Cayley's Formula

$\underline{n=1}$  one tree

$\underline{n=2}$  one tree

$\underline{n=3}$



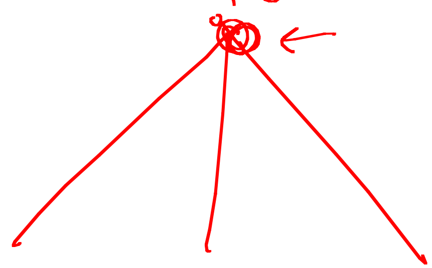
Trees are same, if they have same edge set.

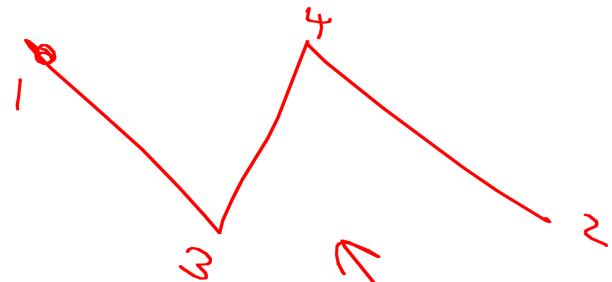
3 trees

$\#$ paths with k vertices $= \frac{1}{2}k!$

$n=4$

4 choices

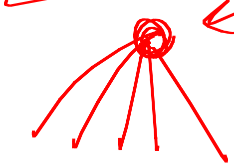


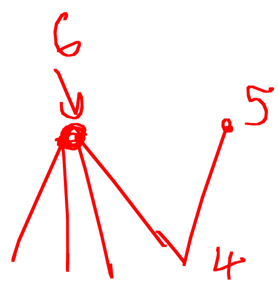


$4!/2 = 12$

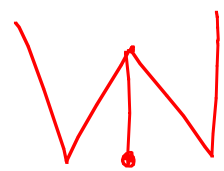
$n=6$

6 choices

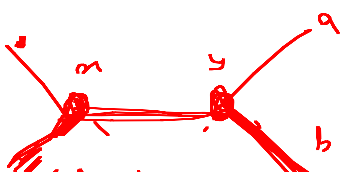




$6 \times 5 \times 4$



$6 \times \frac{1}{2} 5!$

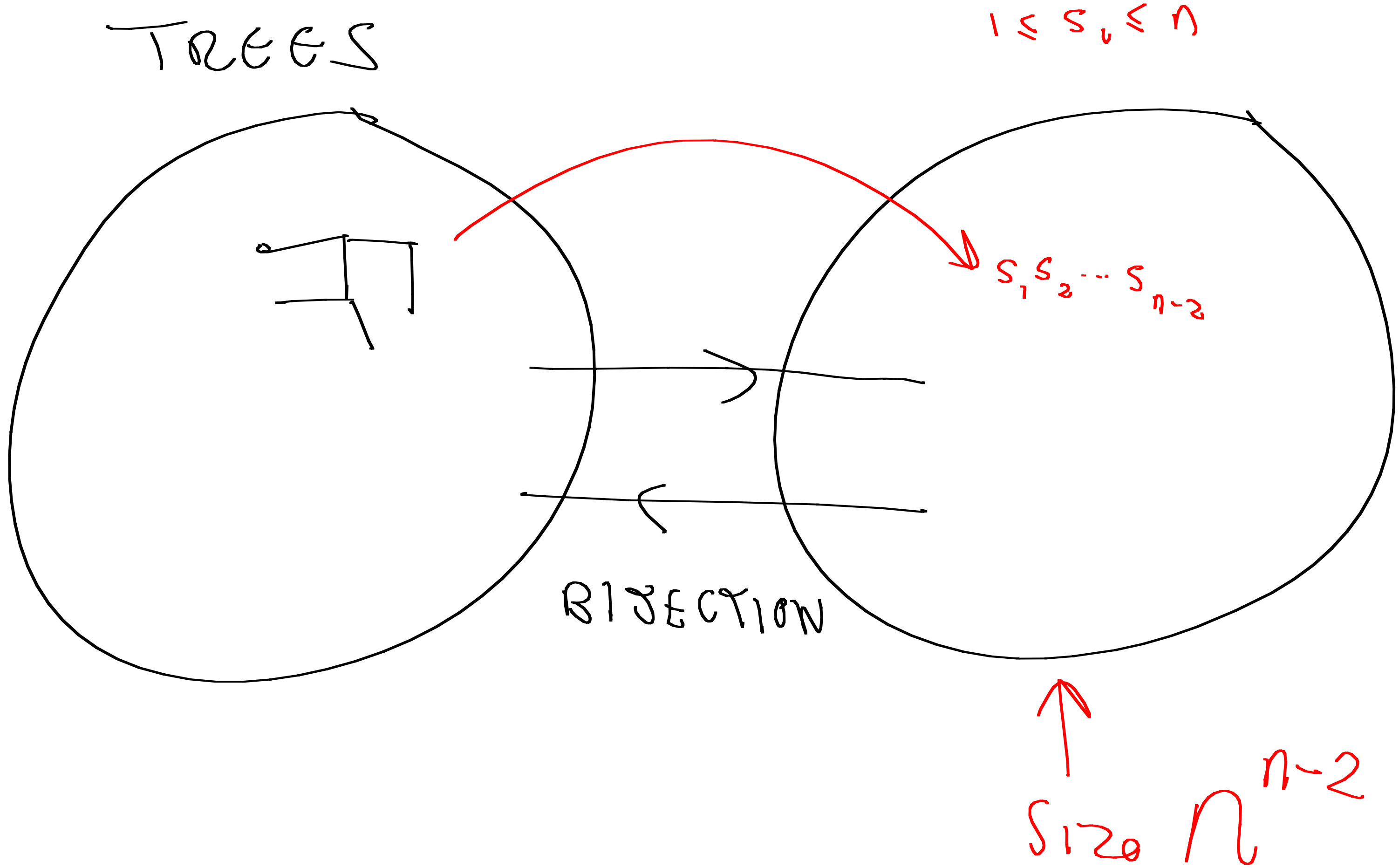


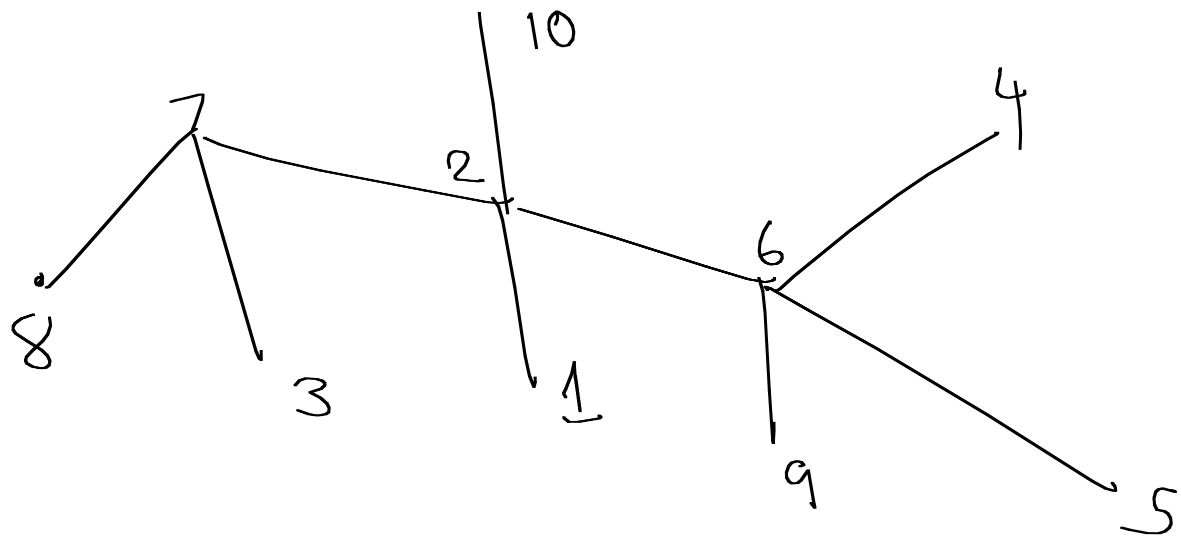
$\binom{6}{2}$ choices for x, y, then $\binom{4}{2}$ choices for a, b

Suppose we have vertex set $\{1, 2, \ldots, n\}$

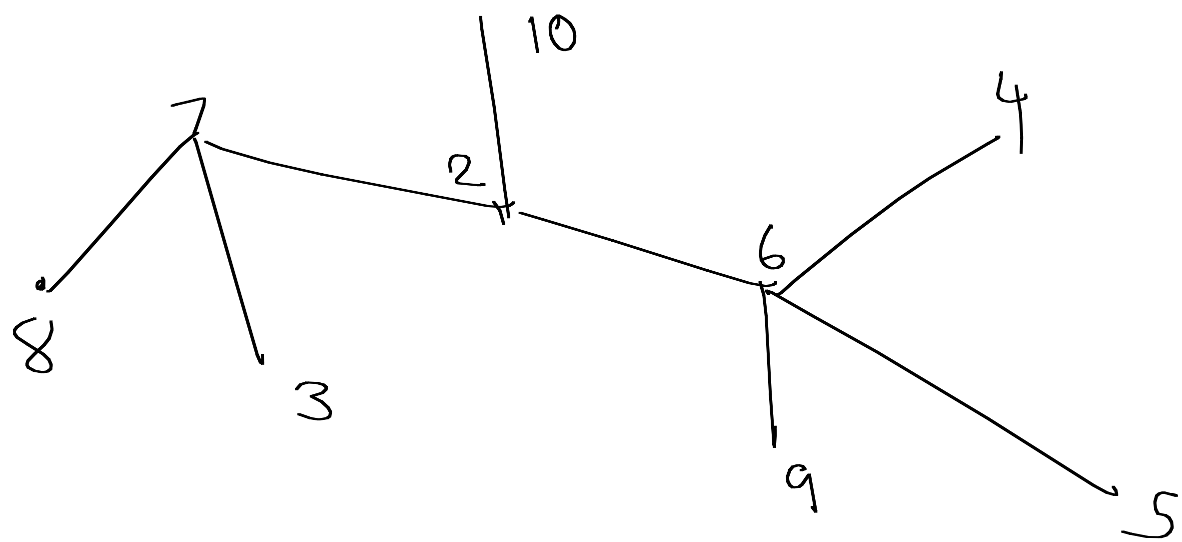
$$\#\text{trees} = n^{n-2}$$

Prufers Correspondences

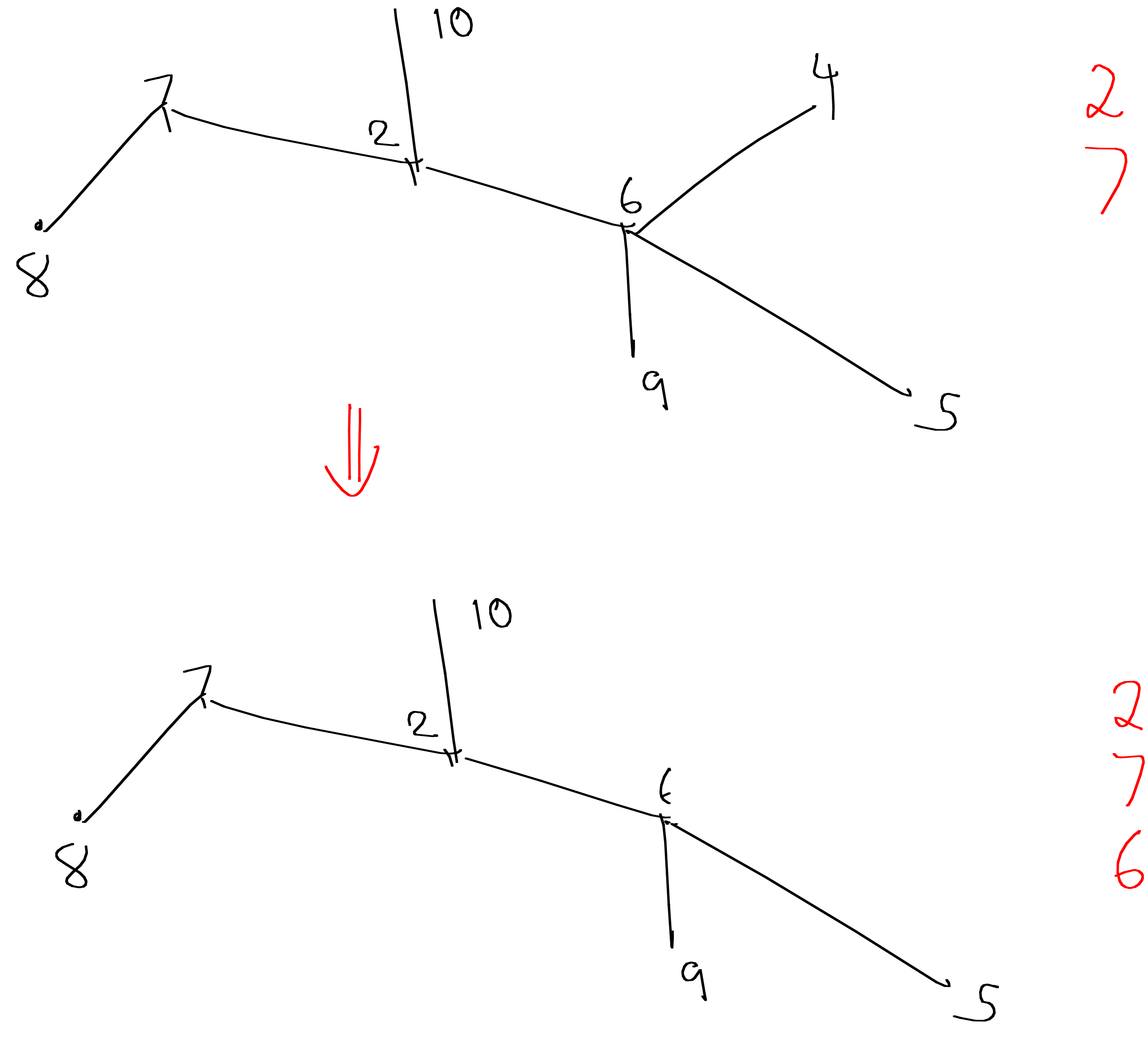




$\Downarrow$



2



10
4
2
6
7
8
9
5
2
7
10
2
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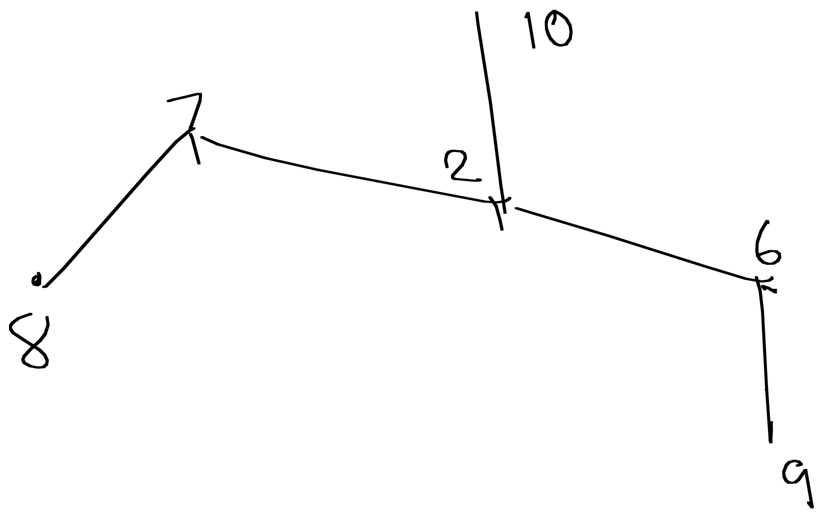
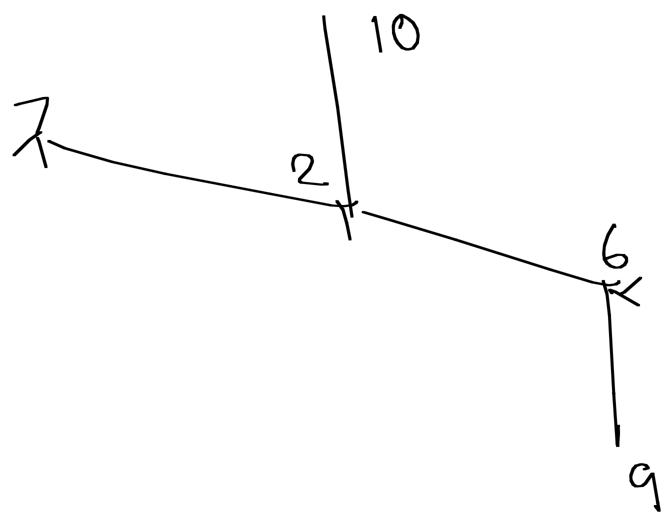
6



2
7
6
6

$\Downarrow$



2
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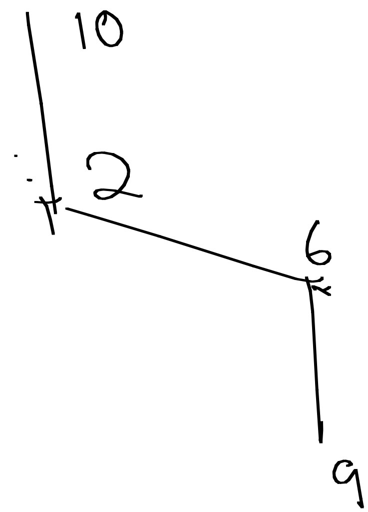
6
7



2
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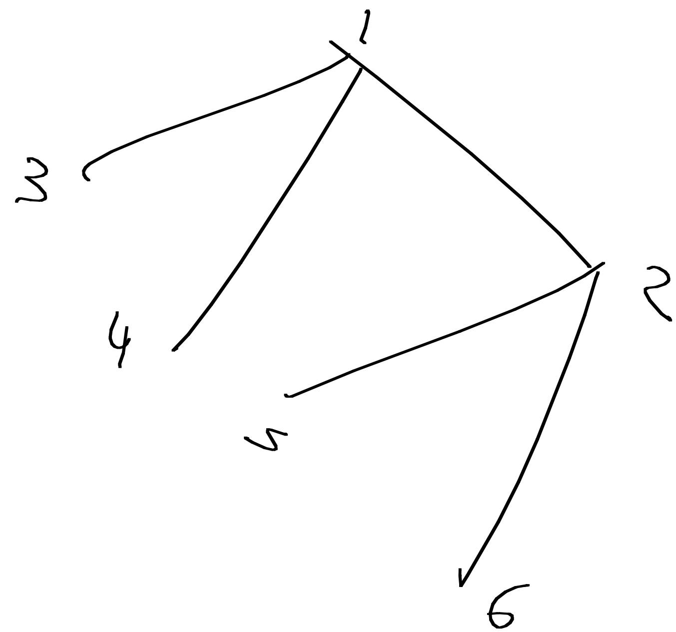

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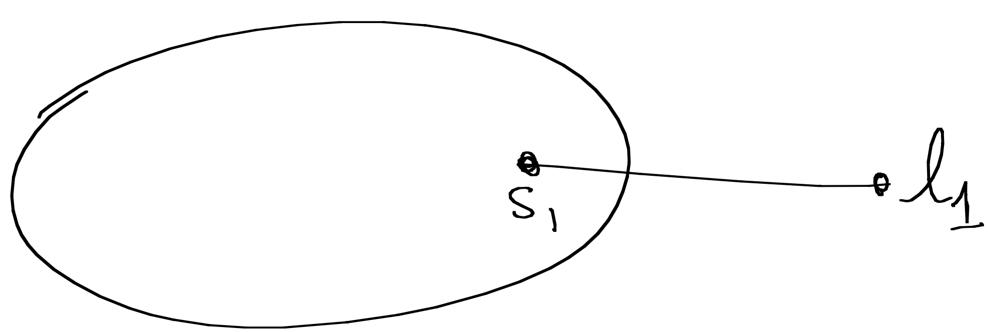
Lemma

$T \longrightarrow$ Sequence

Vertex v appears $d_T(v) - 1$ times.

Sequence : $s_1 s_2 \ldots\ldots$

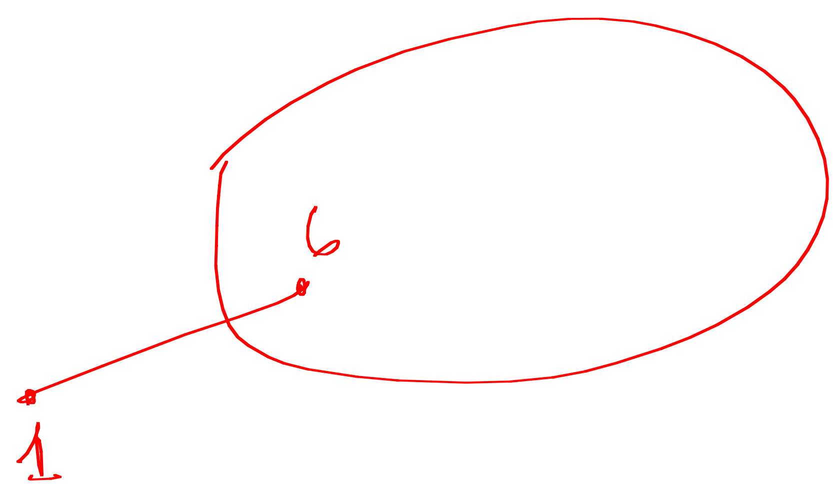
Proof : Induction



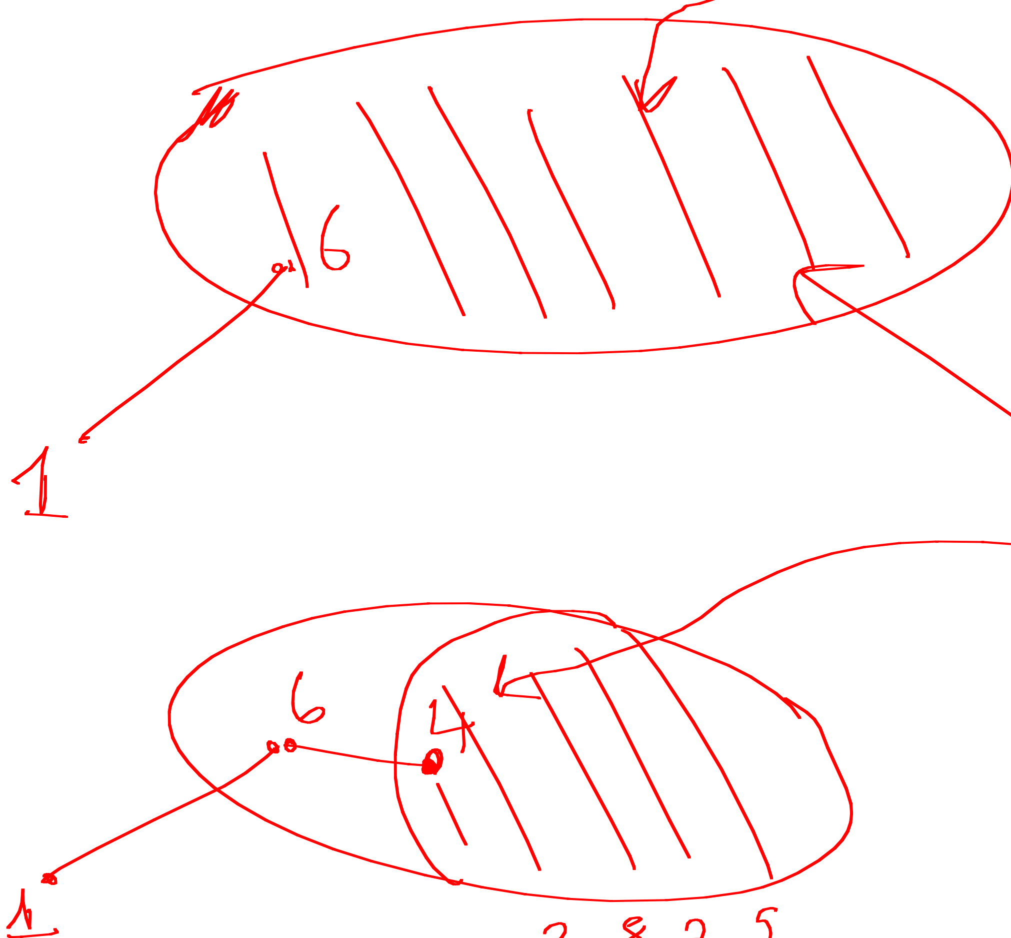
$T \longrightarrow \phi(T)$

tree Sequence.

6 4 3 8 2 5 $n=8$



4 3 8 2 5



Tree from

4 3 8 2 5

(1 not allowed)

$\boxed{2,3,4,6,7,8}$

6 is missing