

21-301 Combinatorics

Homework 9

Due: Wednesday, November 28

1. Consider the following take-away game: There is a pile of n chips. A move consists of removing 5^k chips for some $k \geq 0$. Compute the Sprague-Grundy numbers $g(n)$ for $n \geq 0$.
2. In a take-away game, the set S of the possible numbers of chips to remove is finite. Show that the Sprague-Grundy numbers satisfy $g(n) \leq |S|$ where n is the number of chips remaining.
3. In a take-away game, the set S of the possible numbers of chips to remove is the complement in $\{1, 2, 3, \dots\}$ of a finite set. Show that $g(n) \rightarrow \infty$ with n .