21-301 Combinatorics Homework 8 Due: Friday, November 9

1. Suppose we 2-color the edges of K_n Red and Blue. Let r_i be the Red degree of vertex $i, i \in [n]$. (This is the degree of i in the graph induced by the Red edges).

Show that there are exactly $\binom{n}{3} - \frac{1}{2} \sum_{i=1}^{n} r_i (n-1-r_i)$ mono-chromatic triangles.

Show that if n = 6 then there are at least two monochromatic triangles.

- 2. Suppose that $a, b \ge 1$ and the edges of K_{a+b} are colored Red and Blue. Show that at least one of the following exists: (i) a vertex of Red degree $\ge a$; (ii) a Blue path of length $\ge b$. (Hint: Assume there is no vertex of Red degree a)
- Suppose we 2-color the edges of K_n, n ≥ 3, Red and Blue. Show that at least one of the following exists: (i) a vertex of Red degree ≥ ⌊n/2⌋ − 1; (ii) a blue triangle.