21-301 Combinatorics
Homework 7
Due: Friday, November 2

1. Prove that if n is odd then for any permutation π of $[n]$, the product $P(\pi) = (1 - \pi(1))(2 - \pi(2))...(n - \pi(n))$ is necessarily even.

2. Suppose that $A \subseteq [n]$, $|A| = m$ and $2^m > mn$. Prove that it is possible to select disjoint non-empty subsets $S, T \subseteq A$ whose members have the same sum.

3. Given any sequence of n integers, positive or negative, not necessarily all different, show that some consecutive subsequence has the property that the sum of the members of the subsequence is a multiple of n.