21-301 Combinatorics
Homework 5
Due: Friday, October 12

1. Let k be fixed. Show that there exists a tournament with the following property: For every set S of size k, there exists a disjoint set T of size k such that everyone in T beats everyone in S.

2. A box has m drawers; Drawer i contains g_i gold coins, s_i silver coins and ℓ_i lead coins, for $i = 1, 2, \dots, m$. Assume that one drawer is selected randomly and that two randomly selected coins from that drawer turn out to be gold. What is the probability that the chosen drawer is drawer 1?

3. Let $m = \lfloor (8/7)^{n/3} \rfloor$. Show that there exist distinct sets $A_1, A_2, \dots, A_m \subseteq [n]$ such that for all distinct $i, j, k \in [m]$ we have $A_i \cap A_j \not\subseteq A_k$.