## 21-301 Combinatorics Homework 4 Due: Friday, October 5

1. A permutation  $\pi(1), \pi(2), \ldots, \pi(n)$  is said to be 132 avoiding if there do not exist i < j < k such that  $\pi(j) > \pi(k) > \pi(i)$ . Let  $u_n$  be the number of 132 avoiding permuations. Let  $u_0 = 1, u_1 = 1, u_2 = 2$  and show that for  $n \ge 3$ ,

$$u_n = \sum_{i=1}^n u_{i-1} u_{n-i}$$

(Hint: Count 132 avoiding permutations with  $\pi(i) = n$ .)

Compare this with the recurrence for the number of ways of triangulating a polygon and get an expression for  $u_n$ .

- 2. n distinguishable balls are independently and randomly numbered with m colors, each coloring being equally likely. What is the probability that the number of balls with color 1 is equal to the number of balls with color 2. (The answer is a sum).
- 3. n indistinguishable balls are independently and randomly numbered with m colors, each color being equally likely. What is the probability that the number of balls with color 1 is equal to the number of balls with color 2. (The answer is a sum).