21-301 Combinatorics Homework 3 Due: Friday, September 21

1. Let $A_{n,k}$ be the set of partitions of [n] into k parts, such that each part contains at least two elements. Let $a_{n,k} = |A_{n,k}|$. Show that

$$a_{n,k} = (n-1)a_{n-2,k-1} + ka_{n-1,k}.$$

(Hint: The two terms are the number of partitions in which n appears in a set of size exactly two, at least three, respectively.)

2. A set of k lines in the plane are in general position if no two are parallel and no three have a common intersection. These lines divide the plane into u_k regions bounded by lines. Thus,

$$u_0 = 1; u_1 = 2; u_2 = 4; u_3 = 7; u_4 = 11; u_5 = 16; \dots$$

Find a simple recurrence that this sequence satisfies and find an expression for u_k .

3. Let a_0, a_1, a_2, \ldots be the sequence defined by the recurrence relation

$$a_n + 3a_{n-1} + 2a_{n-2} = 2$$
 for $n \ge 2$

with initial conditions $a_0 = 1$ and $a_1 = 3$. Determine the generating function for this sequence, and use the generating function to determine a_n for all n.