## 21-301 Combinatorics Homework 2 Due: Friday, September 14

1. Use induction to show that

$$\binom{n-1}{k} = \binom{n}{k} - \binom{n}{k-1} + \dots \pm \binom{n}{0}.$$

- 2. In how many ways can 3n distinguishable balls  $b_1, b_2, \ldots, b_{3n}$  be placed in boxes  $B_1, B_2, \ldots, B_n$  so that (i) each box contains three balls and (ii) there does not exist *i* such that box  $B_i$  contains balls  $b_{3i-2}, b_{3i-1}, b_{3i}$ ?
- 3. Find an expression for the size of the set

$$\{(x_1, x_2, \dots, x_m)\} \in Z^m : x_1 + x_2 + \dots + x_m = n \text{ and } 1 \le x_j \le a \text{ for } j = 1, 2, \dots, m\}.$$

[You should use Inclusion-Exclusion and expect to have your answer as a sum.]