21-301 Combinatorics
Homework 10
Due: Wednesday, December 5

1. How many ways are there to 3-color an $n \times n$ chessboard when n is odd. The group G is the usual 8 element group e, a, b, c, p, q, r, s.

2. A necklace is made of 8 beads strung together in a cycle. Find the pattern inventory for the two colourings of the necklace. when the group G is the group of rotations.

3. A necklace is made of 8 beads strung together in a cycle. Find the pattern inventory for the two colourings of the necklace when the group G is the dihedral group D_8.

   The *dihedral* group D_n is the group of symmetries of a regular n-gon under rotations $R_0, R_1, \ldots, R_{n-1}$ and reflections $S_1, S_1 \ldots, S_n$. Here, assuming n is even, the permutations are

   (i) $e = R_0, R_1, \ldots, R_{n-1}$ where R_i is *a rotation through* $i\pi/4$,

   (ii) $S_1, S_1, S_2, \ldots, S_{n/2}$ where S_i is *a rotation about an axis joining two opposite vertices,*

   (iii) $S_{n/2+1}, S_{n/2+2}, \ldots, S_n$ where $S_{n/2+i}$ is *a rotation about an axis joining the midpoints of two opposite edges.*