

21-301 Combinatorics  
Homework 1  
Due: Friday, September 7

1. How many integral solutions of

$$x_1 + x_2 + x_3 + x_4 + x_5 = 100$$

satisfy  $x_1 \geq 3$ ,  $x_2 \geq 10$ ,  $x_3 \geq -3$ ,  $x_4 \geq 6$  and  $x_5 \geq 0$ ?

2. Prove the following equality using a *combinatorial* argument

$$\sum_{i=3}^n \binom{i}{3} \binom{n}{i} = \binom{n}{3} 2^{n-3}.$$

3. A sequence  $a_1 a_2 \cdots a_m$  where  $a_i \in [n]$  is *s-spaced out* if  $a_{i+1} \geq a_i + s$  for  $1 \leq i < m$ .  $s$  is a non-negative integer. Show that the number of such sequences is  $\binom{m+n-s(m-1)-1}{m}$ .