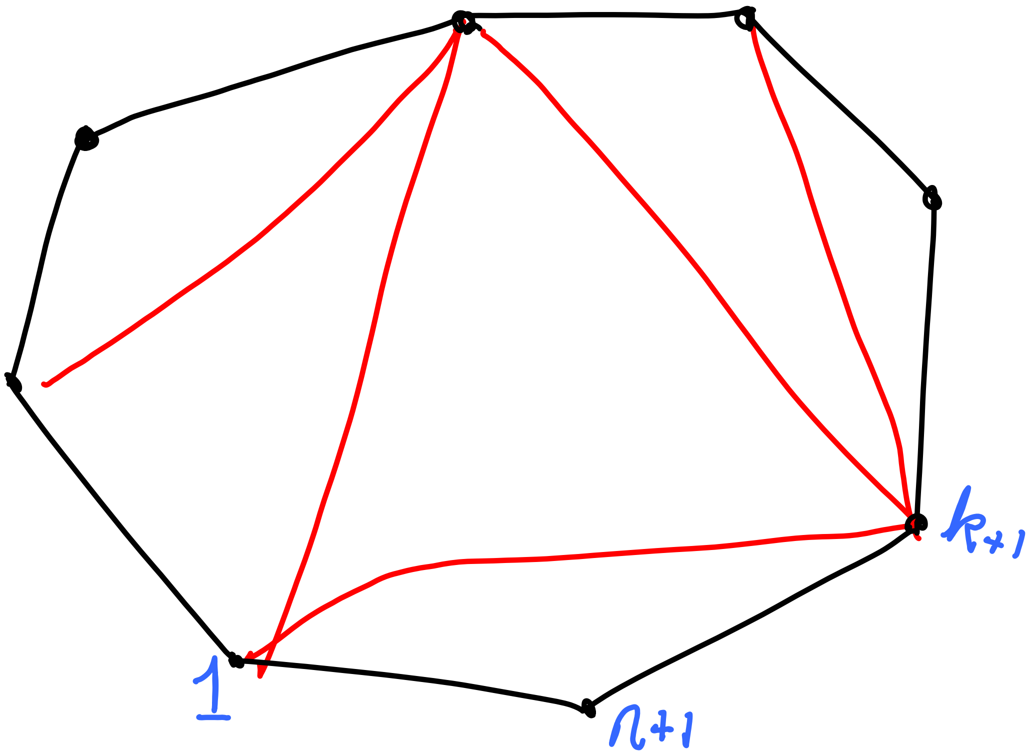
9/21/2007



$a_0 = 0$

$a_1 = 1$

(for convenience)

$a_2 = 1$

$a_n =$ # of triangulations of P_{n+1}

$$= \sum_{k=0}^{n} a_k a_{n-k} \quad \text{for } n \geq 2$$

$$a_n = \sum_{k=0}^{n} a_k a_{n-k} \qquad n \geq 2$$

$$\sum_{n=2}^{\infty} a_n x^n = \sum_{n=2}^{\infty} \left(\sum_{k=0}^{n} a_k a_{n-k} \right) x^n$$

$$a(x) - x$$

$$a(x)^2 = \sum_{n=\cancel{0}\,2}^{\infty} \left(\sum_{k=0}^{n} a_k a_{n-k} \right) x^n$$

$n=0 \qquad a_0 a_0 = 0$

$n=1 \qquad a_0 a_1 + a_1 0 = 0$

$$a(x) - x = a(x)^2$$

$$a(x)^2 - a(x) + x = 0$$

$$a(x) = \frac{1+\sqrt{1-4x}}{2} \quad \text{or} \quad \frac{1-\sqrt{1-4x}}{2}$$

$$a(0) = a_0 = 0$$

$$a(x) = \frac{1}{2} - \frac{1}{2}(1-4x)^{1/2}$$

$$a(x) = \frac{1}{2} - \frac{1}{2}(1-4x)^{1/2}$$

$$= \frac{1}{2} - \frac{1}{2}\sum_{n=0}^{\infty}\binom{1/2}{n}(-4)^n x^n$$

$$\binom{1/2}{n} = \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\cdots\left(\frac{1}{2}-n+1\right)}{n!}$$

$$= \frac{1(-1)(-3)\cdots(3-2n)}{2^n n!}$$

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$$= \frac{(-1)^{n-1}(1\cdot 3\cdot 5\cdot 7\cdots 2n-3)}{2^n n!}$$

$$= \frac{(-1)^{n-1}(2n-2)!}{2^n n!\,(2\cdot 4\cdot \cdots (2n-2))}$$

$$= \frac{(-1)^{n-1}(2n-2)!}{2^{2n-1} n!\,(n-1)!}$$

$$= \frac{(-1)^{n-1}(2n-2)!}{2^{2n-1}\, n!\,(n-1)!}$$

$$\binom{1/2}{n} = \frac{(-1)^{n-1}}{2^{2n-1}\, n} \binom{2n-2}{n-1}$$

$$\frac{1}{2} - \frac{1}{2}\sum_{n=0}^{\infty} \binom{1/2}{n} (-4)^n x^n = \sum_{n=1}^{\infty} \frac{1}{n} \binom{2n-2}{n-1} x^n$$

$$a_n = \frac{1}{n}\binom{2n-2}{n-1}$$

CATALAN NUMBERS