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$$a_n - 6a_{n-1} + 9a_{n-2} = 0 \qquad n \geq 2$$

$$a_0 = 1, \; a_1 = 9$$

$$\sum_{n=2}^{\infty} \left(a_n - 6a_{n-1} + 9a_{n-2}\right) x^n = 0$$

$$\sum_{n=2}^{\infty} a_n x^n - 6 \sum_{n=2}^{\infty} a_{n-1} x^n + 9 \sum_{n=2}^{\infty} a_{n-2} x^n = 0$$

$$a(x) - a_0 - a_1 x \qquad 6x\left[\sum_{n=2}^{\infty} a_{n-1} x^{n-1}\right] \qquad 9x^2 \left[\sum_{n=2}^{\infty} a_{n-2} x^{n-2}\right]$$

$$a(x) - a_0 - a_1 x - 6x\left[\sum_{n=2}^{\infty} a_{n-1}x^{n-1}\right] + 9x^2 \underbrace{\sum_{n=2}^{\infty} a_{n-2}x^{n-2}\Big]}_{a_0 + a_1 x + \cdots} = 0$$

$$a_1 x + a_2 x^2 + \cdots = a(x) - a_0$$

$$a(x) - a_0 - a_1 x - 6x\big(a(x) - a_0\big) + 9x^2 a(x) = 0$$

$$a(x)\left[1 - 6x + 9x^2\right] = a_0 + a_1 x - 6a_0 x$$

$$= 1 + 3x$$

$$a(x) = \frac{1+3x}{1-6x+9x^2}$$

$$\frac{1}{(1-y)^2}$$

$$= 1 + 2y + 3y^2 + \dots + (n+1)y^n + \dots$$

$$= \frac{1+3x}{(1-3x)^2}$$

$$3x \sum_{n=0}^{\infty} (n+1) 3^n x^n$$

$$= \sum_{n=0}^{\infty} (n+1) 3^{n+1} x^{n+1}$$

$$= (1+3x) \sum_{n=0}^{\infty} (n+1) 3^n x^n$$

$$\sum_{n=1}^{\infty} n 3^n x^n$$

$$a_n = [x^n] = (n+1)3^n +$$

$$n 3^n$$

$$= (2n+1)3^n$$

Fibonacci Sequence

$$a_n - a_{n-1} - a_{n-2} = 0 \qquad n \geq 2$$

$$a_0 = a_1 = 1$$

$$\sum_{n=2}^{\infty} (a_n - a_{n-1} - a_{n-2})\, x^n = 0$$

$$a(x) - a_0 - a_1 x \quad - x(a(x) - a_0) \quad - x^2 a(x) = 0$$

$$a(x) = \frac{1}{1 - x - x^2}$$

$$a(x) = \frac{1}{1-x-x^2} = \frac{1}{1-?} + \frac{1}{1-??}$$

$$= -\frac{1}{(\xi_1 - x)(\xi_2 - x)}$$

ξ_1, ξ_2 are roots of $1-x-x^2=0$

$$= \frac{1}{\xi_1 - \xi_2}\left(\frac{1}{\xi_1 - x} - \frac{1}{\xi_2 - x}\right)$$

$$\frac{1}{1-x-x^2} = 1 + (x+x^2) + (x+x^2)^2 + (x+x^2)^3 + \cdots$$

$$= ???$$

$$= \frac{1}{\xi_1 - \xi_2}\left(\frac{1}{\xi_1 - x} - \frac{1}{\xi_2 - x}\right)$$

$$= \frac{1}{\xi_1 - \xi_2}\left(\frac{\xi_1^{-1}}{1 - \frac{x}{\xi_1}} - \frac{\xi_2^{-1}}{1 - \frac{x}{\xi_2}}\right)$$

$$= \frac{\xi_1^{-1}}{\xi_1 - \xi_2}\sum_{n=0}^{\infty} \frac{x^n}{\xi_1^n} - \frac{\xi_2^{-1}}{\xi_1 - \xi_2}\sum_{n=0}^{\infty} \frac{x^n}{\xi_2^n}$$

$$a_n = \frac{1}{\xi_1 - \xi_2}\left[\frac{1}{\xi_1^{n+1}} - \frac{1}{\xi_2^{n+1}}\right]$$

$$a_n - 3a_{n-1} = n^2$$

$a_0 = 1$

$$\sum_{n=1}^{\infty} (a_n - 3a_{n-1}) x^n = \sum_{n=1}^{\infty} n^2 x^n$$

$$a(x) - a_0 - 3x\,a(x)$$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots$$

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + \cdots + nx^{n-1} + (n+1)x^n$$

$$\frac{1}{(1-x)^3} = 2 + 3 \times 2x + 4 \times 3x^2 + \cdots + n(n-1)x^{n-2} + (n+2)(n+1)x^n$$