9/12/2007

n boxes

$B_1 \quad B_2 \quad B_3 \quad B_4 \quad \cdots \quad B_n$

$2n$ balls $b_1 \quad b_2 \quad \cdots \quad b_{2n}$

are placed in the boxes, 2 balls per box.

Q1: How many ways are there of doing this?

$$\# = \frac{(2n)!}{2^n}$$

$$\text{or} = \binom{2n}{3} \times \binom{2n-2}{2} \times \binom{2n-4}{2} \times \cdots \binom{4}{2} \times \binom{3}{2}$$

$$= \frac{(2n)!}{\cancel{(2n-2)!} \times 2} \times \frac{\cancel{(2n-2)!}}{\cancel{(2n-4)!} \times 2} \cdots\cdots$$

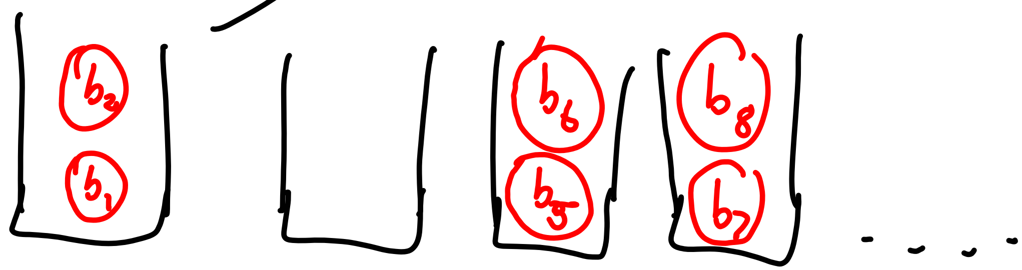
Scrambled : [box i does <u>not</u> get <u>both</u> of b_{2i-1}, b_{2i}] for every i

$A_i = \{$ allocations in which box i gets both b_{2i-1} and b_{2i} $\}$

scrambled allocations

$$= \left|\bigcap_{i=1}^{n} \overline{A}_i\right| = \sum_S (-1)^{|S|} |A_S|$$

$A_{\{1,3,4\}}$



Remaining $2n-6$ balls go into $n-3$ boxes

$$|A_{\{1,3,4\}}| = \frac{(2(n-3))!}{2^{n-3}}$$

In general

$$|A_S| = \frac{(2(n-|S|))!}{2^{n-|S|}}$$

$$\Downarrow$$

I.E. formula

Problème des Ménages

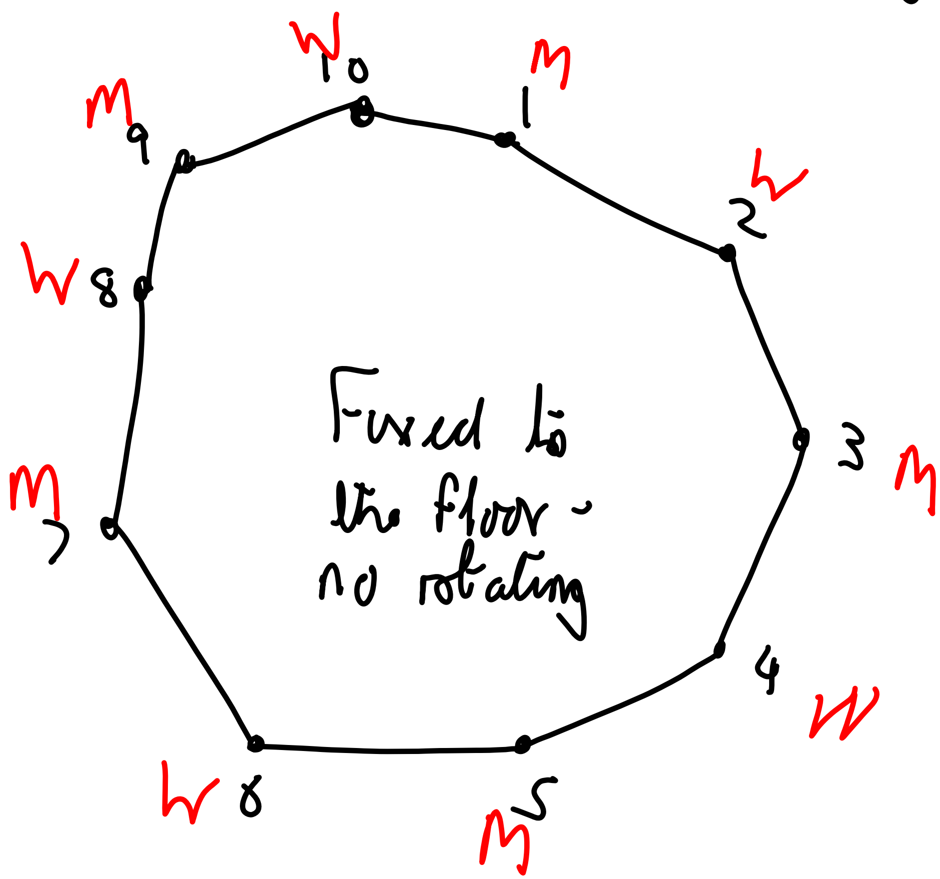


Table has $2n$ seats

n men

$m_1, m_2, \dots m_n$

n women

$w_1, w_2, \dots w_n$

But: m_i, w_i are not allowed to sit next to each other.

$2(n!)^2$ if there are no restrictions.

not: couple 1 sit together

and

not: couple 2 sit together

and

$$M_n = \left| \bigcap_{i=1}^{n} \overline{A_i} \right|$$

$A_i := \{$ arrangements where m_i sits next to $w_i \}$

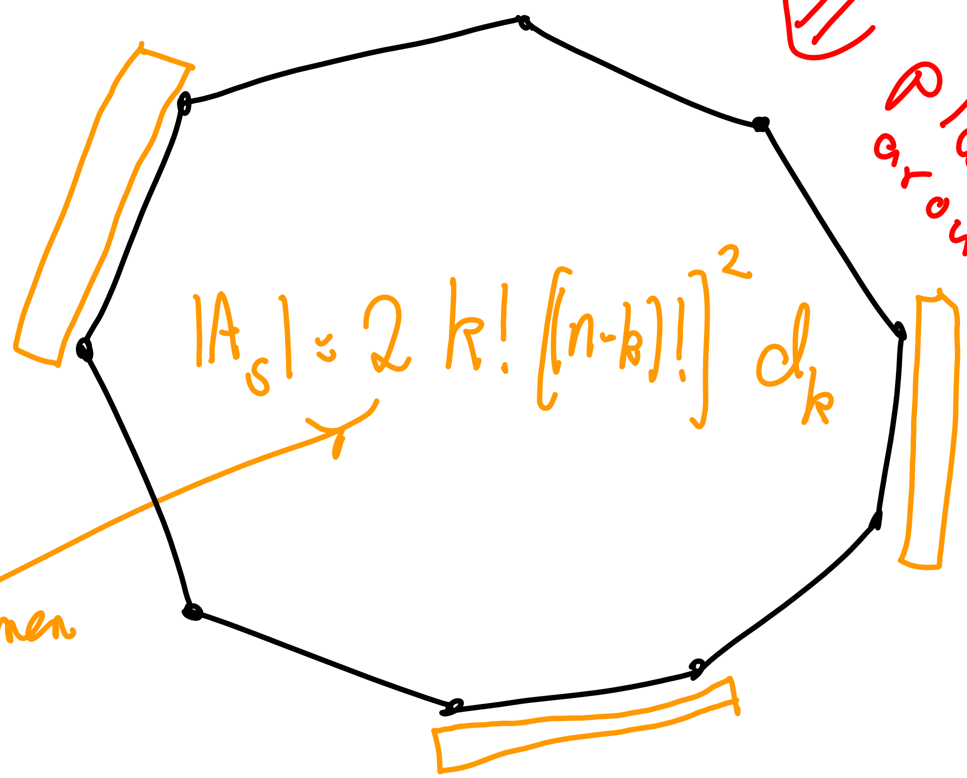
$|A_S| = ??$

$|S| = k$

$i \in S$

$m_i \mid w_i$ . . .

\# ways is d_k $\Rightarrow$ place the k around the table

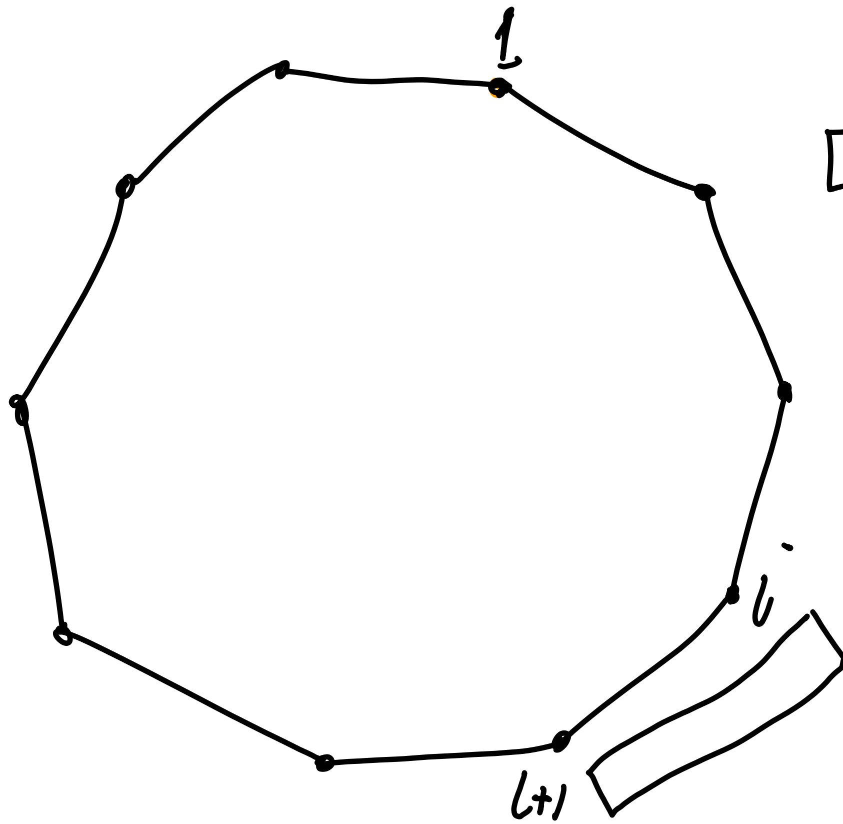


$|A_S| = 2\, k!\, [(n-k)!]^2\, d_k$

Where the Men\Women go

$$a_k = \frac{2n}{k}\binom{2n-k-1}{k-1}$$

(i)



(i) choose i and place one ▭ at (i, i+1)

(ii) Think there are 2n-2 -(k-1) places to choose k-1 from.