9/05/2007

$$\{a, a, a, b, b, c, c, d, d, d\} \neq \{a, b, c, d\}$$

Set $\{a, b, c\}$ 6 permutations

abc
acb
bac
bca
cab
cba

Multi-set $\{a, b, b\}$

abb
bab
bba

permutations of $\{3\times a, 4\times b, 5\times c\}$

Write all permutations as follows:

Look at set

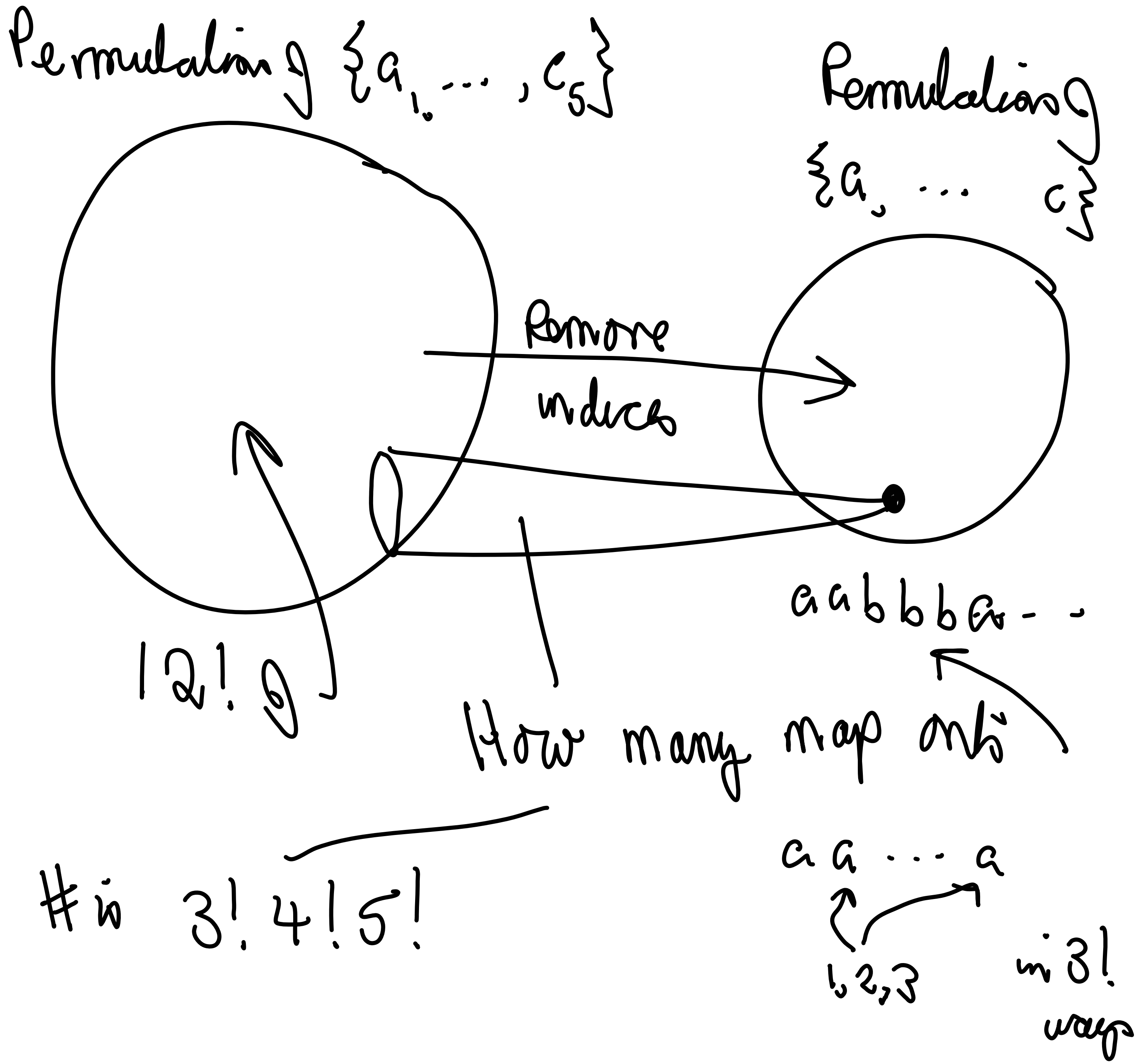
$$\{a_1, a_2, a_3, b_1, b_2, b_3, b_4, c_1, c_2, c_3, c_4, c_5\}$$

Write down all 12! permutation

$$a_1\, a_2\, a_3\, b_1\, b_3\, c_5\, b_2\, c_4\, c_3\, c_2\, c_1\, b_4$$

erase indices

$$a\, a\, a\, b\, b\, c\, b\, c\, c\, c\, c\, b$$


Permutations {a₁ ... , c₅}
Permutations {a, ... c}
Remove indices
12! g
aabbba - -
How many map onto
is 3! 4! 5!
a a ... a
1, 2, 3
in 3! ways

permutations of $\{3 \times a, 4 \times b, 5 \times c\}$

is
$$\frac{12!}{3!\,4!\,5!}$$

In general
permutations of $\{m_1 \times 1, m_2 \times 2, \ldots, m_n \times n\}$

is
$$\frac{(m_1 + m_2 + \cdots + m_n)!}{m_1!\, m_2! \cdots m_n!}$$

Multinomial Coefficient

$$\binom{m}{a_1, a_2, \dots, a_n} = \frac{m!}{a_1! \, a_2! \cdots a_n!}$$

Here $a_1 + \cdots + a_n = m$

Binomial

$$(x_1 + x_2)^m = \sum_{a_1 + a_2 = m} \binom{m}{a_1} x_1^{a_1} x_2^{a_2}$$

$$(x_1+x_2+x_3)(x_1+x_2+x_3)(x_1+x_2+x_3)(x_1+x_2+x_3)$$

$$= \dots + \binom{4}{2,1,1} x_1^2 x_2 x_3 + \dots$$

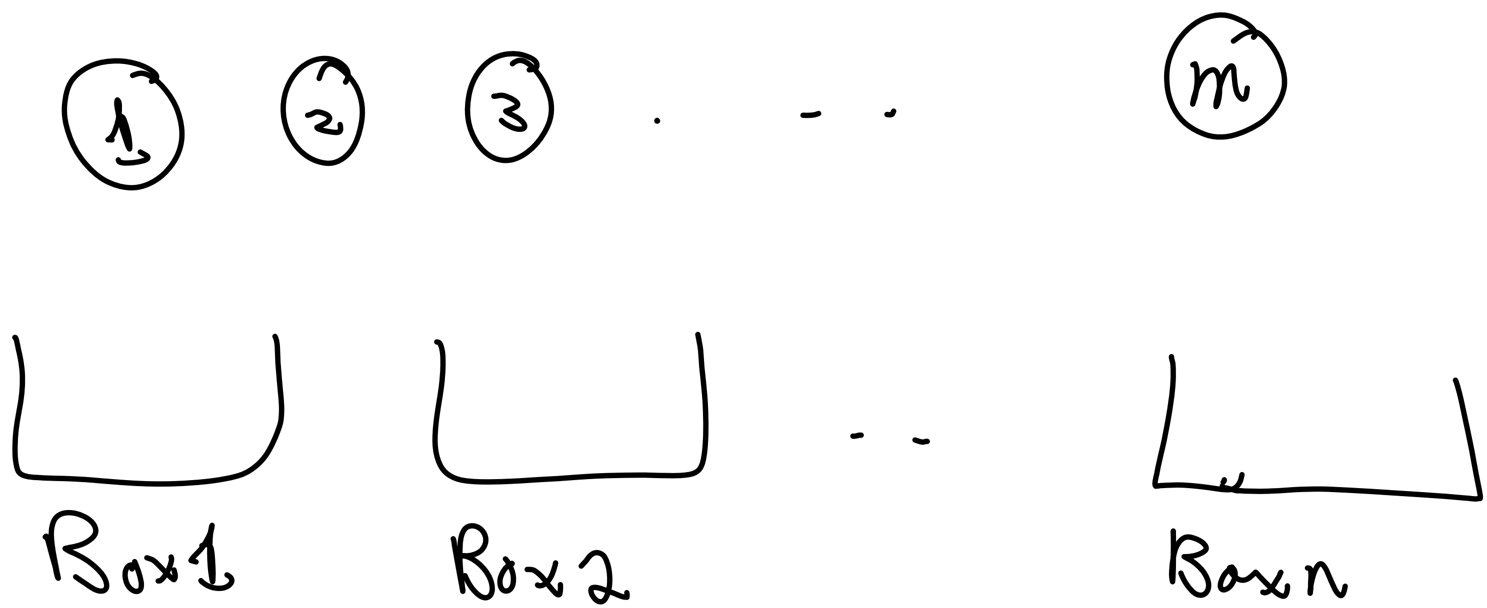
$$\{2 \times x_1,\ 1 \times x_2,\ 1 \times x_3\}$$

every permutation yields a 1 for

$$x_1 x_1 x_2 x_3 + \dots + x_1 x_2 x_1 x_3 + \dots$$

Balls in boxes

m distinguishable balls



Put balls into boxes: # ways $= n^m$

Suppose I have to put b_i balls into Box i.

$$\# \text{ways} = \binom{m}{b_1, b_2, \ldots, b_n}$$

$m = 7$, $n = 3$.

$\overset{2}{b_1 \times 1}$, $\overset{2}{b_2 \times 2}$, . .

\# ways = \# permutations

3 1 3 2 1 3 2

$b_1 = 2$

$b_2 = 2$

$b_3 = 3$