8/31/2007

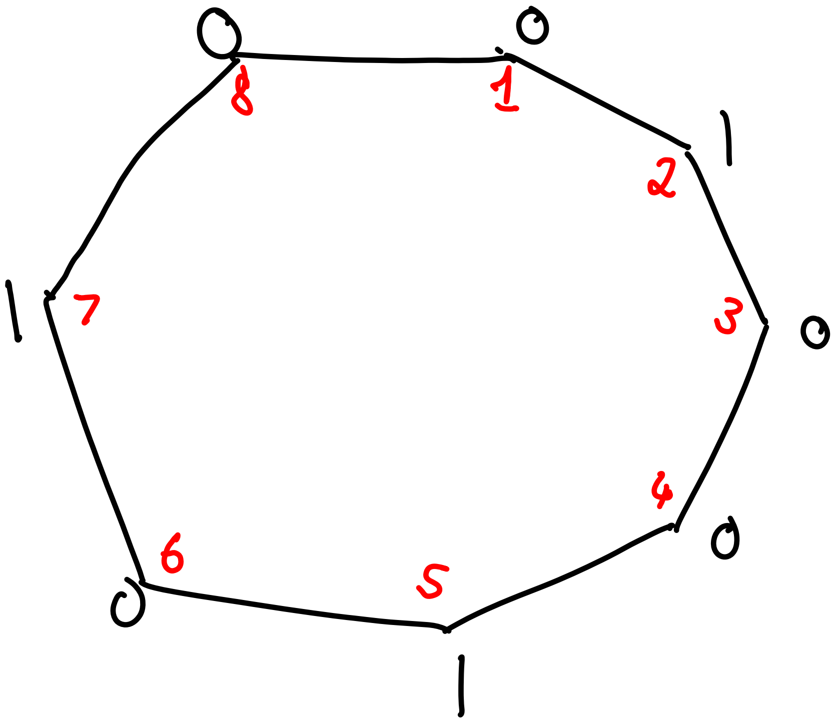
# of ways of doing this is

$$n \binom{n-k-1}{k-1}$$

Each pattern comes up exactly k times in this construction.

$$\#\text{patterns} = \frac{1}{k} \times n \binom{n-k-1}{k-1}$$

This process produces $n\binom{n-k-1}{k-1}$ patterns, not all the same.



	a_1	a_2	a_3
2 :	2	1	2
5 :	1	2	2
7 :	2	2	1

$k=3$

$$\binom{n}{r} = \binom{n}{n-r}$$

$$\frac{n!}{r!\,(n-r)!} = \frac{n!}{(n-r)!\,(n-(n-r))!}$$

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

$$\frac{n!}{k!\,(n-k)!} + \frac{n!}{(k+1)!\,(n-k-1)!}$$

$$= \frac{n!}{k!\,(n-k-1)!}\left[\frac{1}{n-k} + \frac{1}{k+1}\right]$$

$$= \frac{n!}{k!\,(n-k-1)!} \cdot \frac{n+1}{(n-k)(k+1)}$$

Fix k: use induction on n.

Base Case: $n = k$

$$\binom{k}{k} = \binom{k+1}{k+1}$$

LHS RHS

Induction Step: $n \geq k$

$$\underbrace{\binom{k}{k} + \cdots + \binom{n}{k}}_{\text{induction}} + \binom{n+1}{k} = \binom{n+1}{k+1} + \binom{n+1}{k}$$

Pascal

$$= \binom{n+2}{k+1} \quad \Delta$$

S = all $(k+1)$-subsets of $[n+1]$

$S_m = \{ X \in S : \max S = m+1 \}$

(i) $S = S_{k+1} \cup S_{k+2} \cup \cdots \cup S_n$ partition

(ii) $|S| = |S_{k+1}| + |S_{k+2}| + \cdots + |S_n|$

(iii) $|S_m| = \binom{m}{k}$,

Vandermonde's Identity

$$\overset{A}{\underbrace{x\;x\;x\;x\;x\;x\;x\;x}_{m}}\;\Big|\;\overset{B}{\underbrace{x\;x\;x\;x\;x\;x\;x\;x\;x\;x\;x\;x\;x\;x}_{n}}$$

$$S = \binom{[m+n]}{k}$$

$$S_1 = \{X \in S : |X \cap A| = r,\ |X \cap B| = k-r\}$$

$S_0, S_1, \dots, S_k$ partition S

$$|S_r| = \binom{m}{r}\binom{n}{k-r}$$

$$(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r$$

Coefficient of x^r = # ways of

choose x, r times

$$\underbrace{(1+x)(1+x)(1+x)\cdots(1+x)(1+x)}_{n \text{ products}}$$

Induction

$$(1+x)^{n+1} = \underbrace{(1+x)\sum_{r=0}^{n}\binom{n}{r}x^r}$$

Coefficient of x^{r+1}

$$\binom{n}{r+1} + \binom{n}{r} = \binom{n+1}{r+1}.$$

$\underline{x = 1}$

$$(1+1)^n = \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n}$$

$$\shortparallel$$

$$2^n$$

$\underline{x = -1}$

$$(1-1)^n = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots$$

$$\shortparallel$$

$$0$$

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$$

differentiate
w.r.t x

$$n(1+x)^{n-1} = \sum_{k=0}^{n} \binom{n}{k} k x^{k-1}$$

$x=1$

$$n2^{n-1} = \sum_{k=1}^{n} k\binom{n}{k}$$