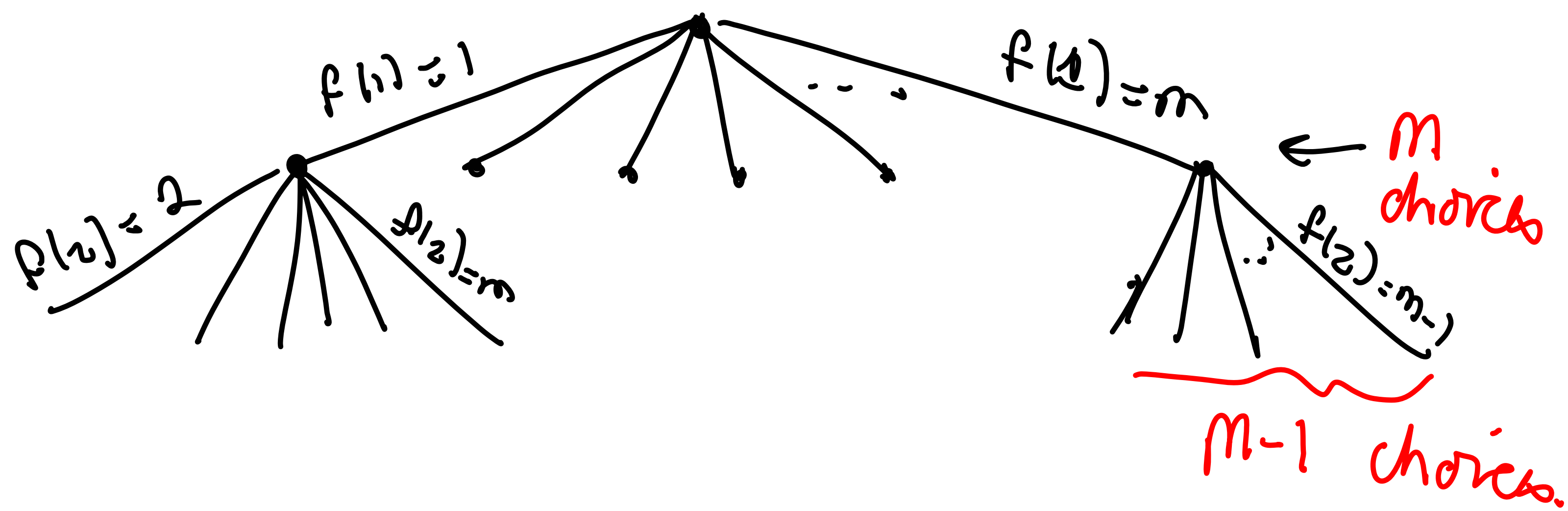
8/29/2007

$\Phi_{1-1}(m,n) = \#$ of 1-1 functions.

$= m(m-1)\cdots(m-n+1)$.



$$\begin{matrix} a_1 & a_2 & \cdots & a_n \\ \uparrow & \uparrow & \cdots & \uparrow \\ f(1) & f(2) & & f(n) \end{matrix}$$

\# of sequences of length n,

where $a_i \in [m]$ and $a_i \neq a_j \ \forall i \neq j$

$= m(m-1) \cdots (m-n+1).$

If $m = n$ then $a_1, a_2, \cdots a_n$ is a permutation of $1, 2, \cdots, n$

(Each no. in $1, 2, \cdots, n$ appears exactly once.)

$\#$ permutation $= n(n-1) \cdots 1$

$= n!$

Binomial Coefficients

$$\binom{n}{k} \quad n \text{ choose } k$$

$$= \frac{n!}{k!\,(n-k)!} = \frac{n(n-1)\ldots(n-k+1)}{k(k-1)\ldots\ldots 1}$$

X is a set.

$\binom{X}{k}$ is the collection of subsets of X of size k.

$$\left|\binom{X}{k}\right| = \binom{|X|}{k}$$

Choose a sequence $a_1, a_2, \ldots a_k$ from $[n]$.

$$\# \text{ways} = n(n-1)\cdots(n-k+1)$$

We can choose a set S of size k and order it, in $k!$ ways.

$$n(n-1)\cdots(n-k+1) = \left|\binom{[n]}{k}\right| \times k!$$

$$\left|\binom{[n]}{k}\right| = \frac{n(n-1)\cdots(n-k+1)}{k!}$$

$$= \binom{n}{k}.$$

$$S(m,n) = \{ i_1, i_2, \ldots, i_n \in \mathbb{Z}^+ : i_1 + i_2 + \cdots + i_n = m \}$$

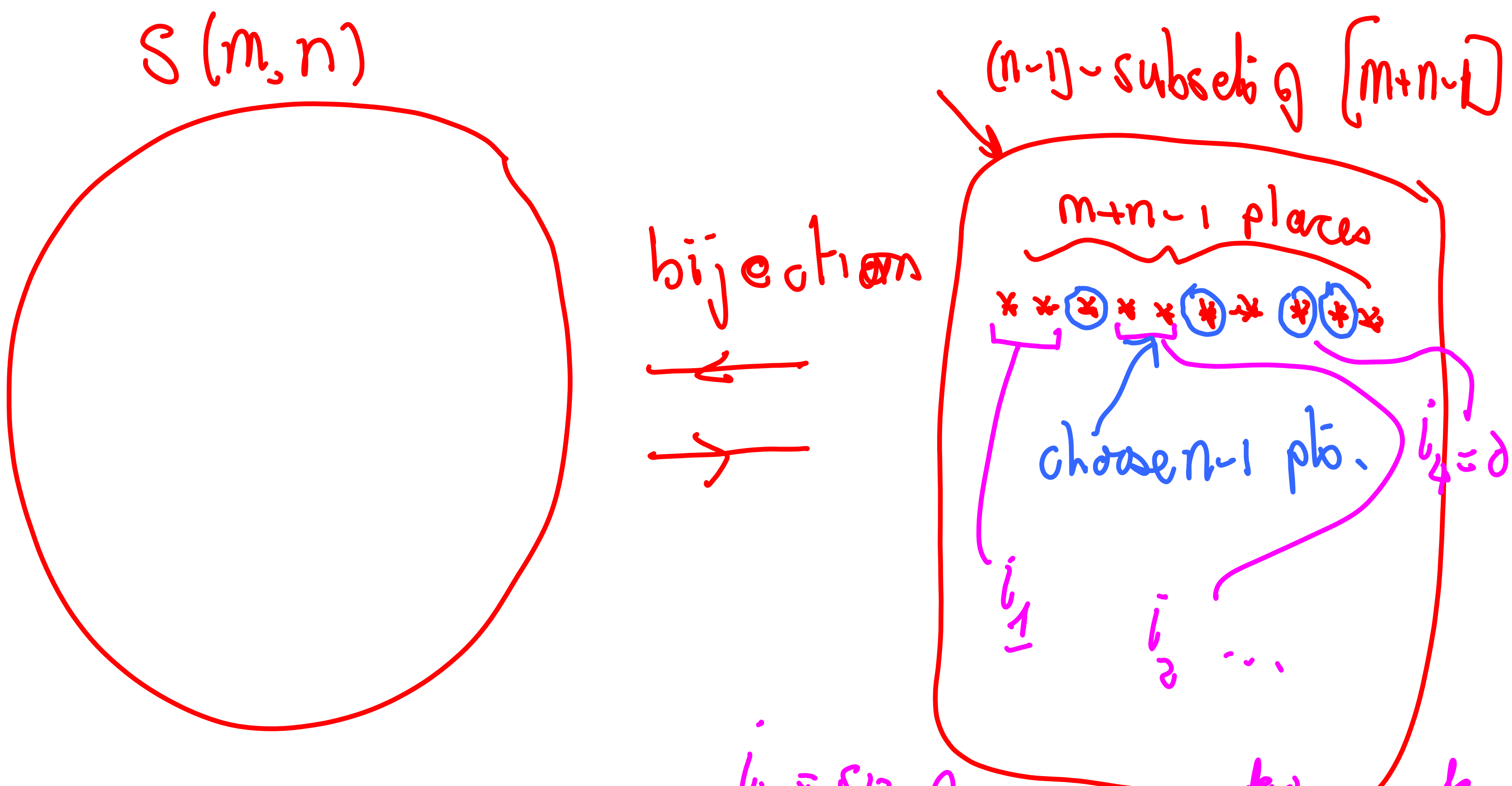
non-negative integers

$n = 3$

$m = 4$

$$S(4,3) = \left\{ \begin{array}{ll} 4,0,0 & \vdots \\ 3,1,0 & \vdots \\ 3,0,1 & \\ 2,2,0 & 0,0,4 \\ \vdots & \end{array} \right\}$$

$$|S(m,n)| = \binom{m+n-1}{n-1}$$



i_k = size of gap between $k-1$ ⊛ k ⊛

$$S(m,n)^* = \{(i_1, i_2, \dots, i_n) \in S(m,n) :$$
$$i_1 \geq 1, i_2 \geq 1, \dots, i_n \geq 1\}$$

$$j_t = i_t - 1 \quad , t = 1, 2, \dots, n$$

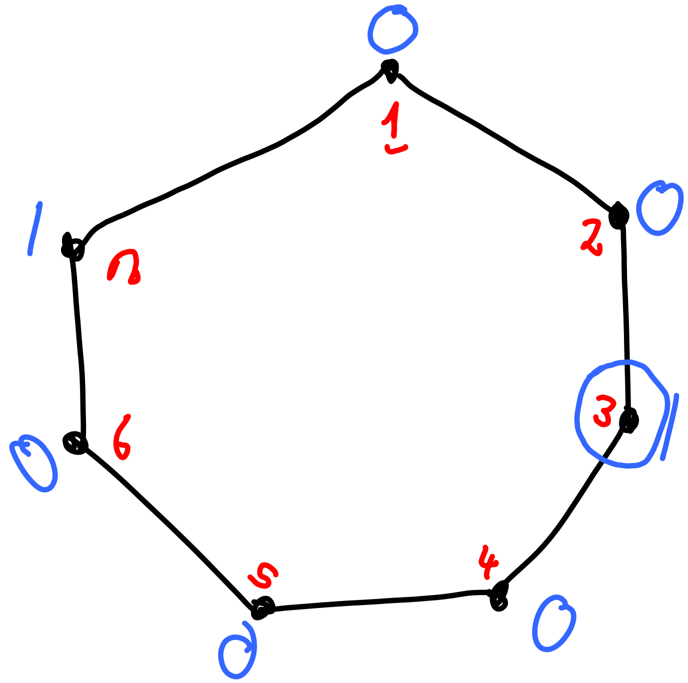
$$\#(\underline{j}) = \#(\underline{i})$$

$$j_t \geq 0 , \quad t = 1, 2, \dots, n$$

$$j_1 + \dots + j_n = m - n$$

$$\#(\underline{j}) = \binom{m-n+n-1}{n-1} = \binom{m-1}{n-1}.$$

$k = 2$

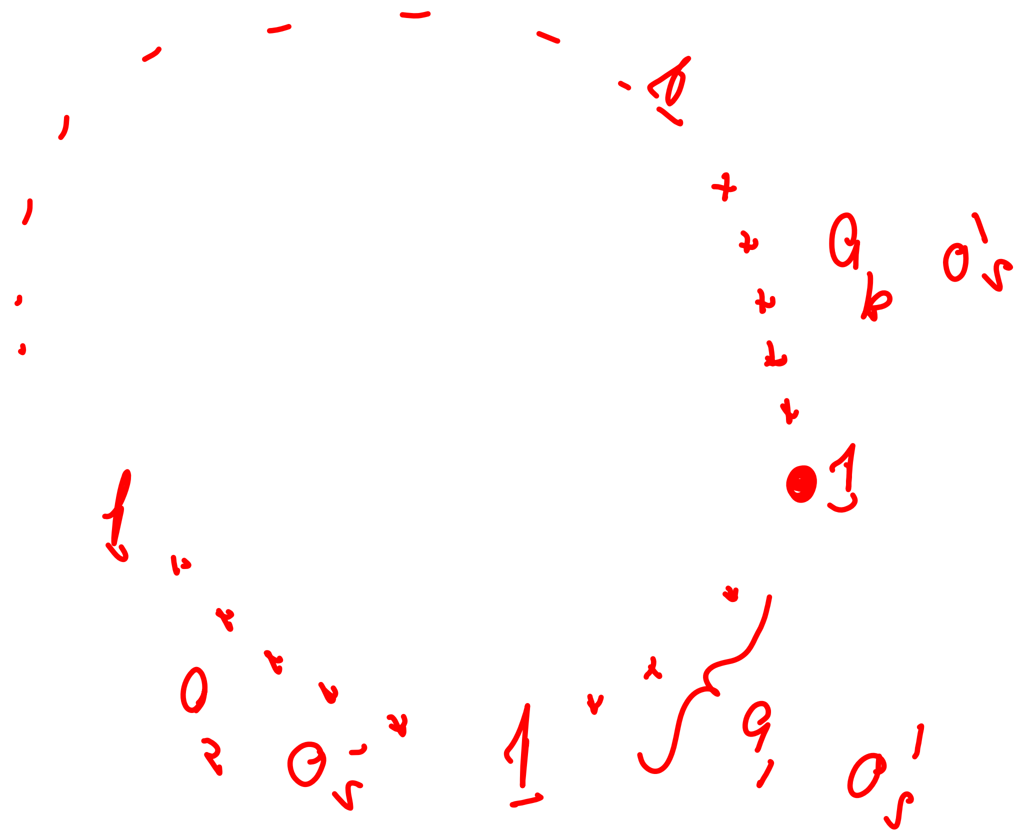


Place k 1's around the polygon.

To construct all patterns:

(i) Choose a place and put a 1 : n choices

(ii) Having chosen the 1, 1 put in the rest.



\# of ways of choosing $a_1, a_2, \dots a_k$?

(a) $a_1 \geq 1, \dots, a_k \geq 1$

(b) $a_1 + \dots + a_k = n-k$

\# choices for $a_1 \dots a_k$ is $\binom{n-k-1}{k-1}$

# of ways of doing this is

$$n \binom{n-k-1}{k-1}$$

Each pattern comes up exactly k times in this construction.

$$\#\text{patterns} = \frac{1}{k} \times n \binom{n-k-1}{k-1}$$