8\27\2007

$\phi(m,n)$ = # functions from

$[n] = \{1, 2, \cdots, n\}$

to

$[m]$

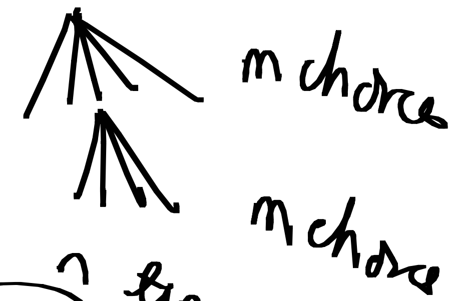
$$= \Big| \underbrace{[m]^{[n]}}_{A} \Big|$$

$|A|$ = # elements in A

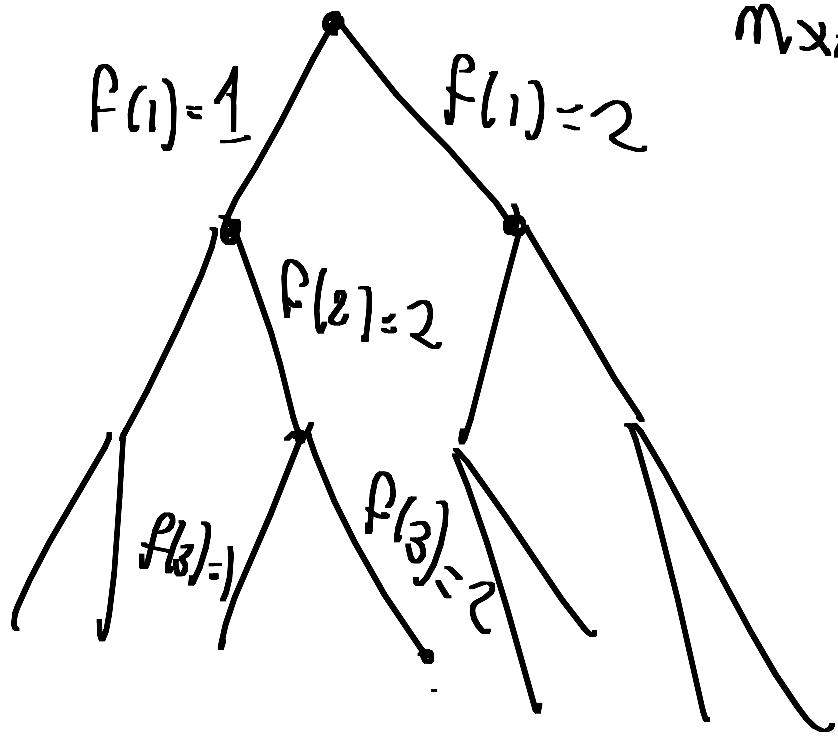
$X^Y = \{$fnc from $Y \to X\}$

$$\phi(m, n) = m^n$$

$n = 3, \quad m = 2$



$$\overbrace{m \times m \times \cdots \times m}^{n \text{ times}} = m^n$$



Proper Proof $\phi(m,n) = m^n$

By induction on n.

Base Case: $n = 0$

fns from $\emptyset \longrightarrow [m]$: $\underline{1}$

$m^0 = \underline{1}$ ✓

Inductive Step:

functions $[n+1] \longrightarrow [m]$

(i) Choose $f(n+1)$ — m ways

(ii) Choose rest — $\phi(m, n)$ ways.

$$\# \text{ choice} = m \times \phi(m, n)$$
$$= m \times m^n$$
$$= m^{n+1}$$

$\square$

$\phi(m, n)$ = # of sequence

$x_1 x_2 \cdots x_n$

where $x_i \in [m]$

$f(i) = x_i$

$f(1) = 1 \quad f(2) = 4 \quad f(3) = 2 \quad f(4) = 1$

1 4 2 1

# of subsets of $[n]$ is 2^n

$= \phi(2, n)$

$A \subseteq [n]$

$n = 7$

$A = \{1, 3, 4\}$

A: f_A (indicator function)

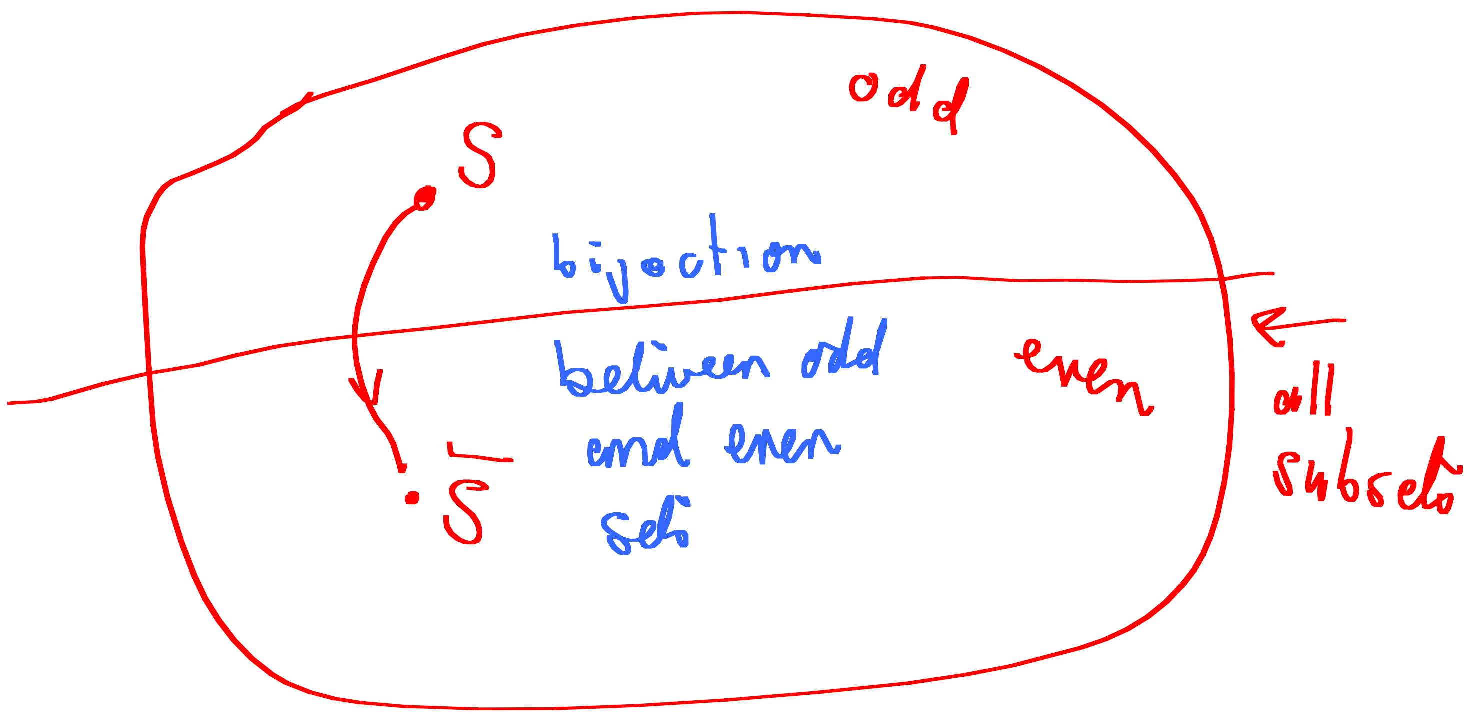
$f_A(1) = 1 \quad f_A(2) = 0 \quad f_A(3) = 1 \quad \ldots$

$1\ 0\ 1\ 1\ 0\ 0\ 0 \leftarrow 2^n$ sequences.

For any n, # of odd subsets of $[n]$ = # even subsets.

Case 1: n is odd

$$\bar{S} = [n] \setminus S$$



n even

No good