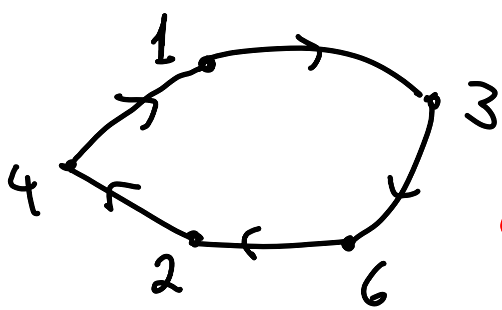
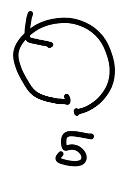
11\30\2007

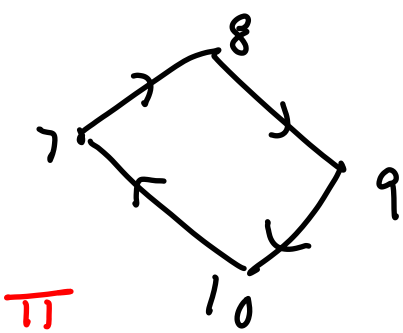
Cycles of a permutation:

$n = 10$

i	1	2	3	4	5	6	7	8	9	10
$\pi(i)$	3	4	6	1	5	2	8	9	10	7

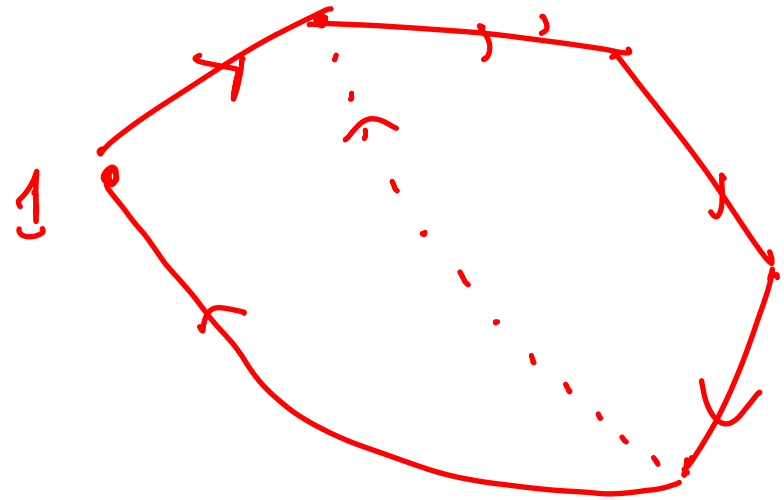






cycles of π

Every π is the union of disjoint cycles



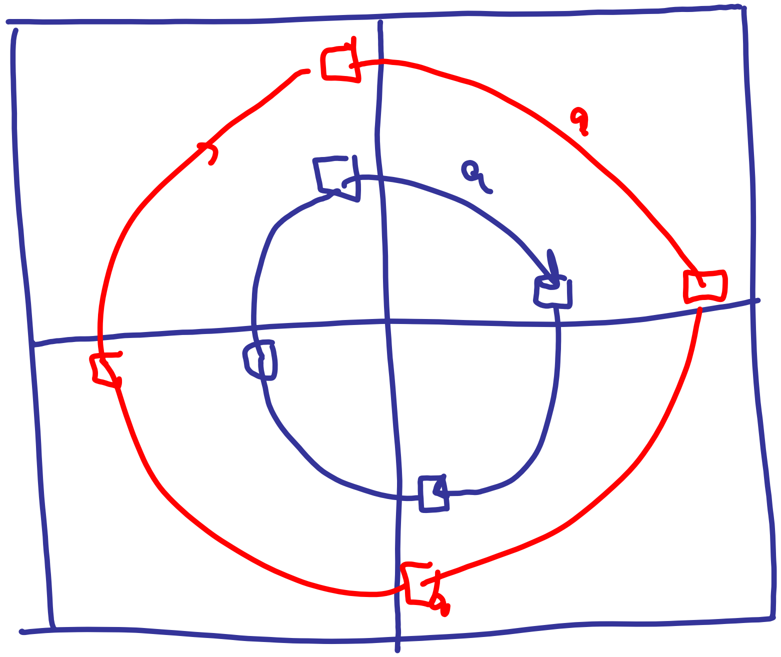
Repeat

We will argue that if there are
k colors then $|Fix(g)| = k^{\#\text{cycles in } g}$

$n \times n$

n even

Fix (a)



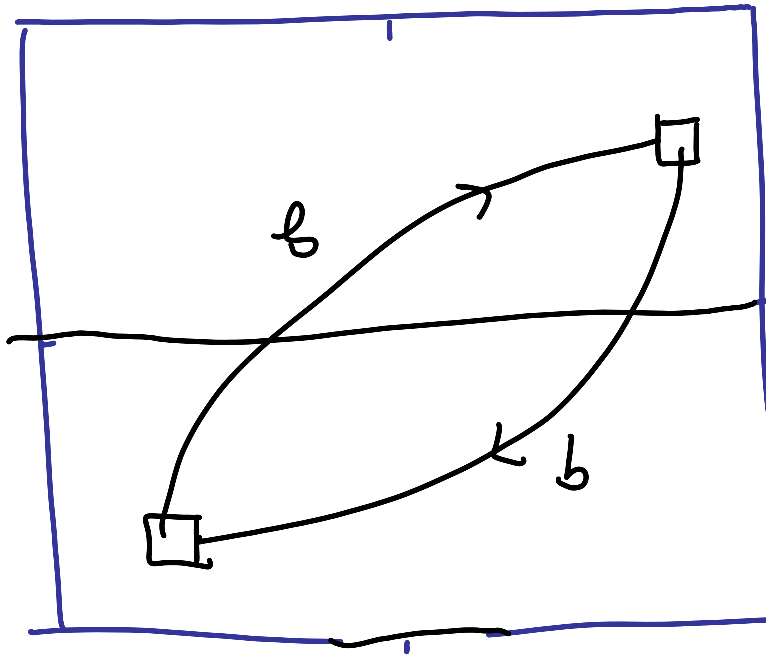
$|X| = 2^{n^2}$

$\# \text{orbits} = \frac{1}{8}\Big(\underset{e}{2^{n^2}} + \underset{a}{2^{n^2/4}} \times \quad \underset{b}{} \quad \underset{c}{} \quad \underset{p}{} \quad \underset{q}{} \quad \underset{r}{} \quad \underset{s}{} \Big)$

$n \times n$

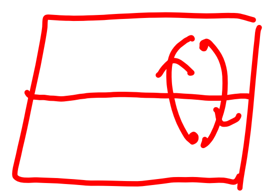
n even

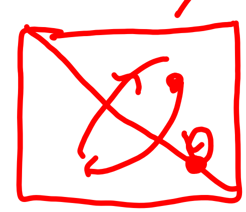
Fix(b)



$$|X| = 2^{n^2}$$

$$\#\text{orbits} = \frac{1}{8}\Big(\underset{e}{2^{n^2}} + \underset{a}{2^{n^2/4}} + \underset{b}{2^{n^2/2}} + \underset{c}{2^{n^2/4}} + \underset{p}{\ } \quad \underset{q}{\ } \quad \underset{r}{\ } \quad \underset{s}{\ }\Big)$$





Polya Theorem

We can compute # distinct colorings of $n \times n$ chessboard.

Now we answer questions like.

How many distinct colorings have k Red and $n^2 - k$ blue squares.

D = doman

- sectors of disk
- squares of chessboard
- $\vdots$

$C = \{\text{colors}\}$

$X = \{\text{colorings}\} = \{x : D \to C\}$

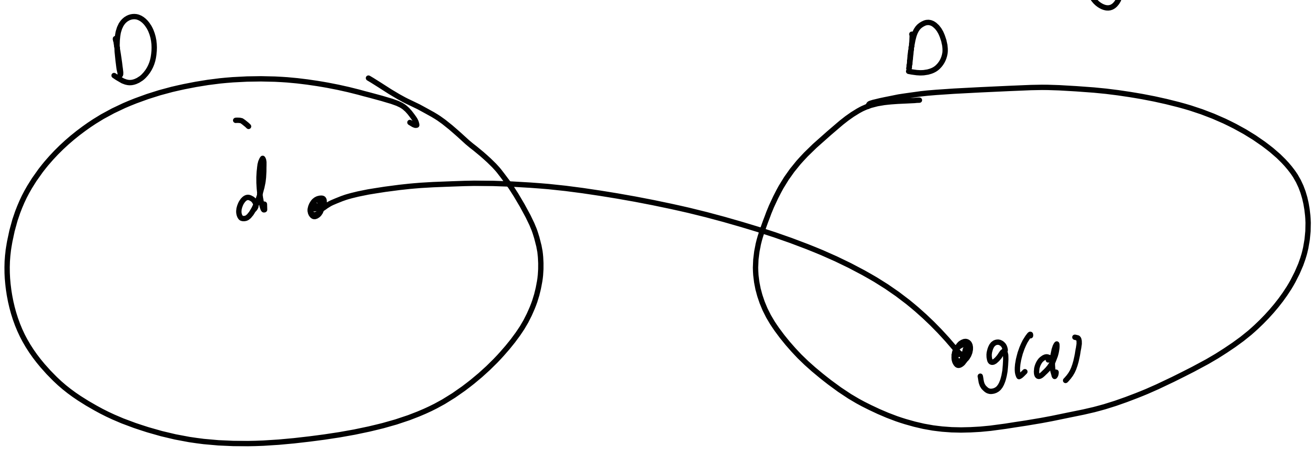
$x(d)$ is the color of $d \in D$

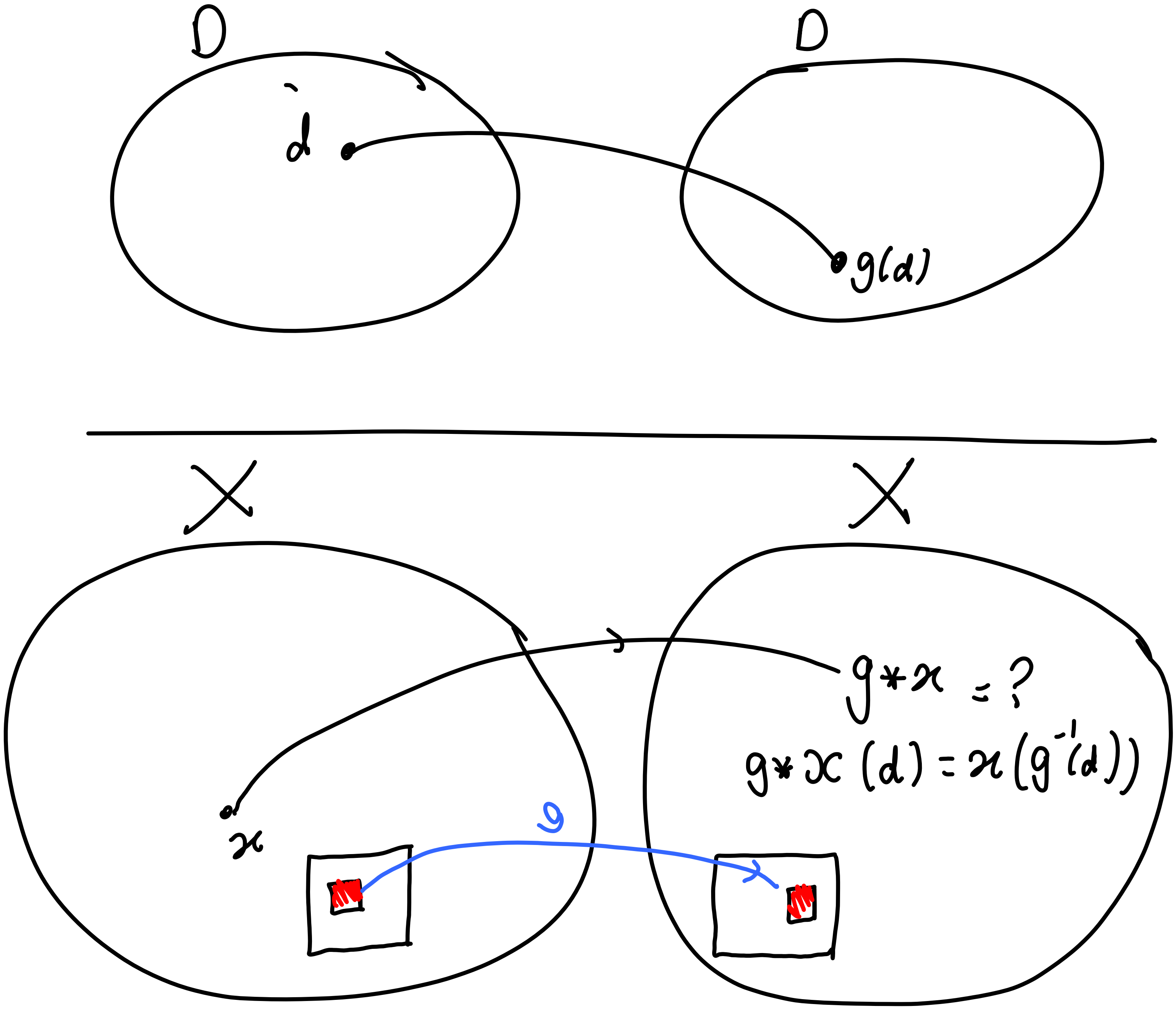
G is a group acting on D.

G is a group acting on D.

We show how to extend to a permutation g X.

Given g & d we can compute $g(d)$





D
d
D
g(d)
X
X
g*x = ?
g*x (d) = x(g^{-1}(d))
x
g