11\28\2007

For any $x \in X$

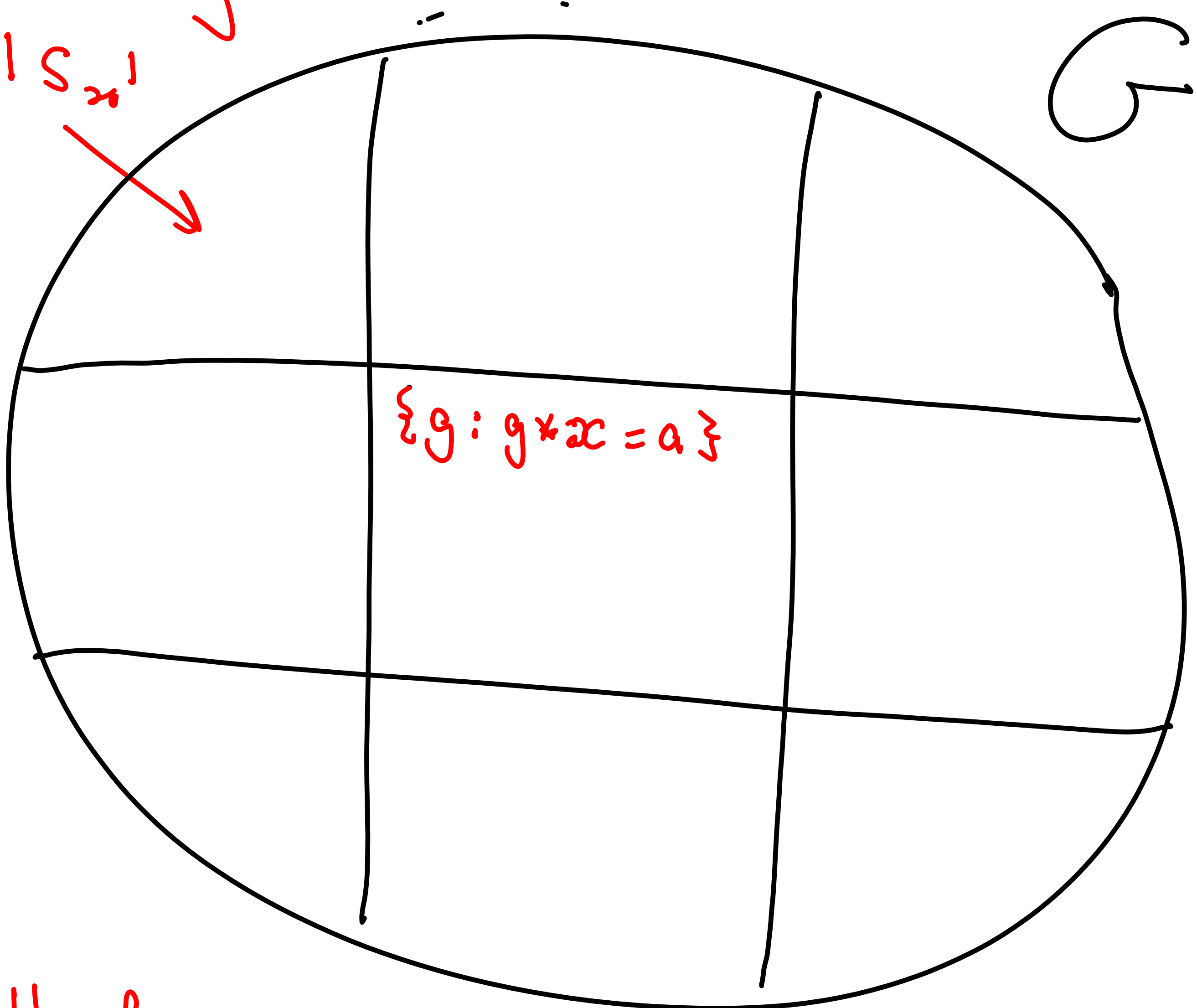
$$|O_x||S_x| = |G|.$$

Fix $x \in X$

Define equivalence relation $\sim$ on G

by $g \sim h$ iff $g * x = h * x$

Each class is of size $|S_x|$



\# of equivalence classes $= \#a\text{'s} = |O_x|$

How many g's make $g * x = a$

Fix one item g_1

$$g_1 * x = a$$

$$\left[\begin{array}{l} g_1 * x = a \\ x = g_1^{-1} * a \end{array}\right.$$

$$(g_1^{-1} \circ g_2) * x = g_1^{-1} * (\underbrace{g_2 * x}_{a}) = x$$

$$g_1^{-1} \circ g_2 \in S_x$$

$$g_2 \in g_1 \circ S_x$$

$$\{g : g * x = a\}$$

$g_1 \cdot$

(i) $g_2 \in \quad \Rightarrow \quad g_2 \in g_1 \circ S_x$

(ii) Suppose $h = g_1 \circ s$ where $s \in S_x$

$$h_1 * x = (g_1 \circ s) * x = g_1 * (s * x) = g_1 * x = a$$

(iii) $s_1, s_2 \in S_x$ & $g_1 \circ s_1 = g_1 \circ s_2$

$$\Rightarrow \quad s_1 = s_2 .$$

Theorem

$$|O_x||S_x| = |G|.$$

$$\# \text{orbits} = \sum_{x \in X} \frac{1}{|O_x|}$$

$$= \sum_{x \in X} \frac{|S_x|}{|G|}$$

$$= \frac{1}{|G|} \sum_{x \in X} |S_x|.$$

$\{1,2,3,\dots,10\}$

$= \{1,2,3\}$

$\cup \{4,5\}$

$\cup \{6,7,8,9\}$

$\cup \{10\}$

$\sum_{x \in X} =$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

$+ \frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

$+ 1 = 4$

$$\mathrm{Fix}(g) := \{x : g * x = x\}$$

$g \in G$.

$$\#\text{orbits} = \frac{1}{|G|} \sum_{g \in G} |\mathrm{Fix}(g)|$$

$$\begin{array}{cc} & g \\ x & \left[\begin{array}{c} \mathbb{1}_{g * x = x} \end{array}\right] \end{array} \quad \#1\text{'s in row } x = |S_x|$$

#1's in column $= |\mathrm{Fix}(g)|$