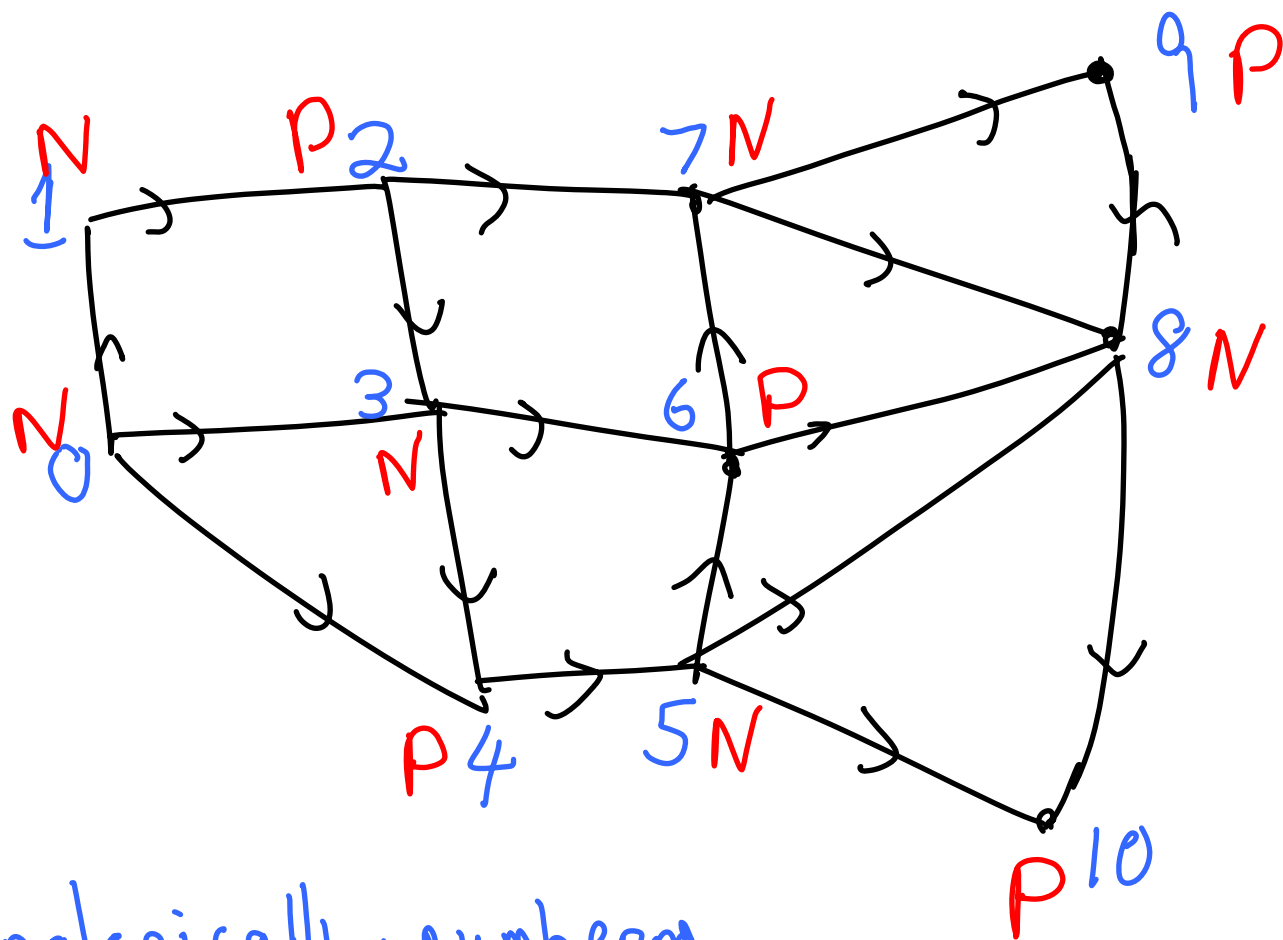


11/16/2007

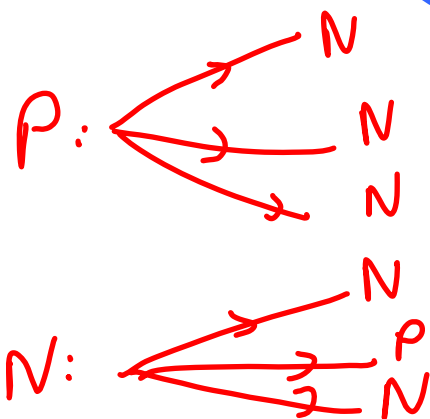
Positions of game:

P-positions: losing vertex for
next player

N-positions: winning vertex for next
player.



Topologically numbering



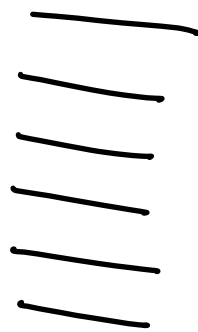
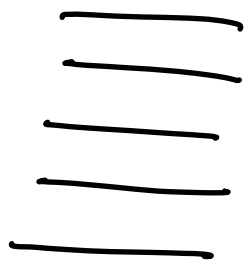
Sums of Games

Games $G_1, G_2, \dots, G_p$

$$G = G_1 \oplus G_2 \oplus \dots \oplus G_p$$

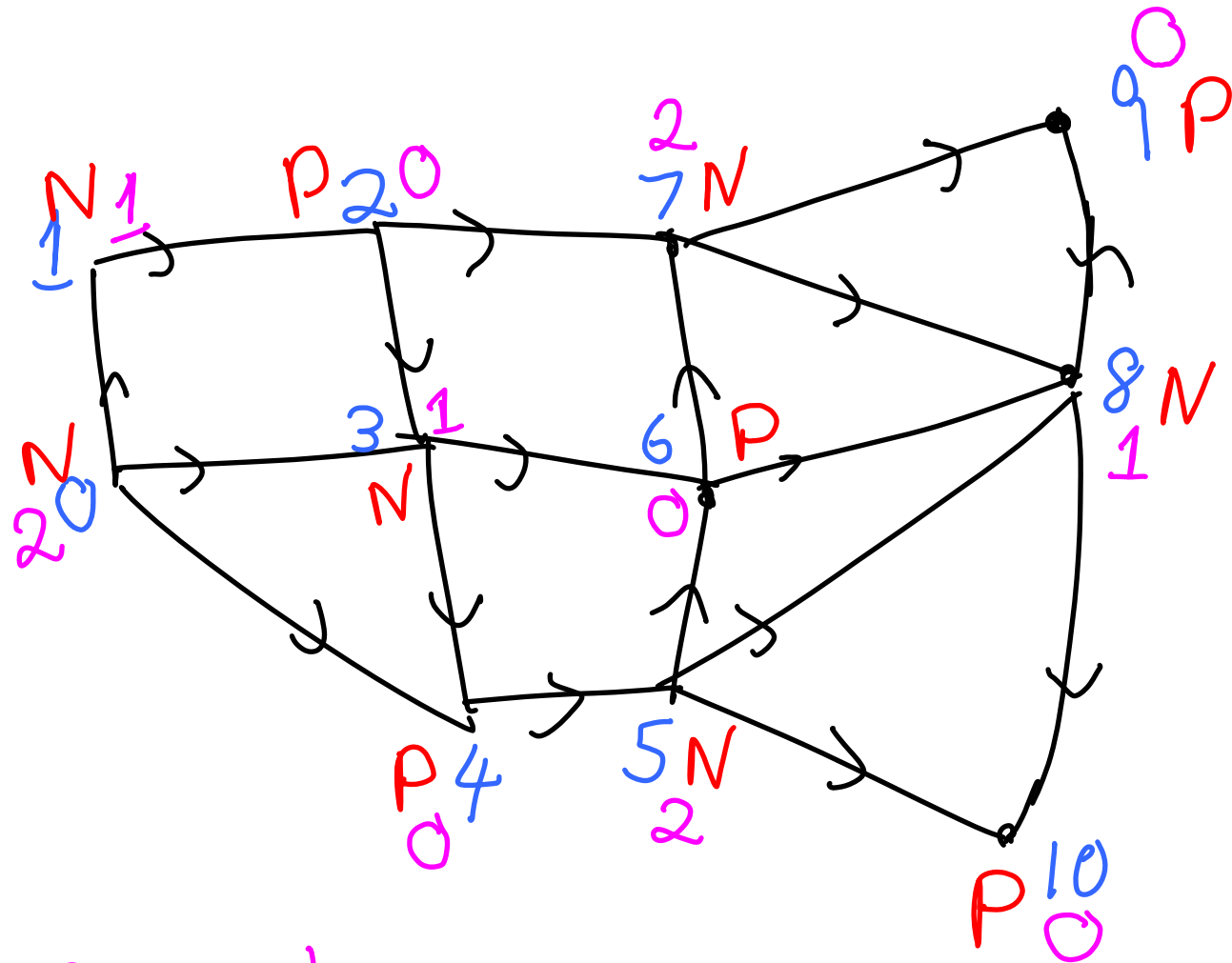
A move consists of

- (1) Choose i
- (2) Move in G_i



Num :

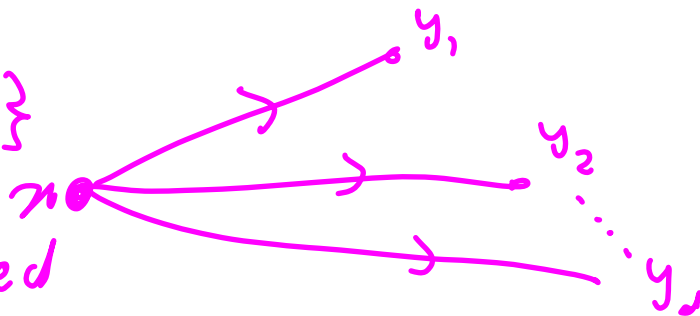
$$G_1 \oplus G_2 \oplus G_3$$



SG-numbers

$$mex = \max \{y_1, y_2, \dots, y_d\}$$

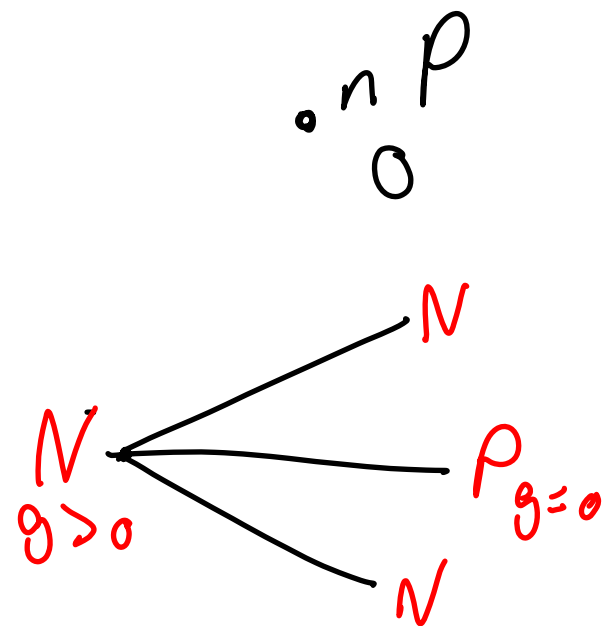
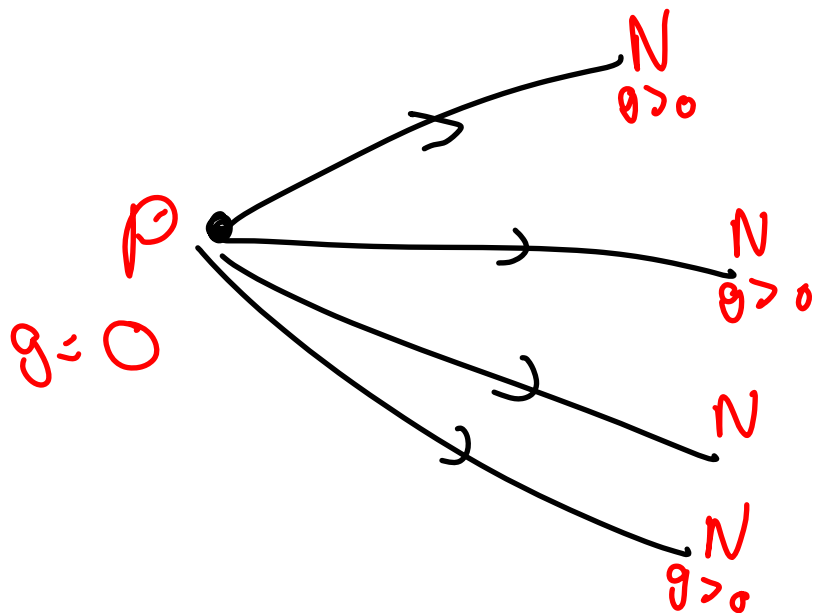
↑
minimum excluded

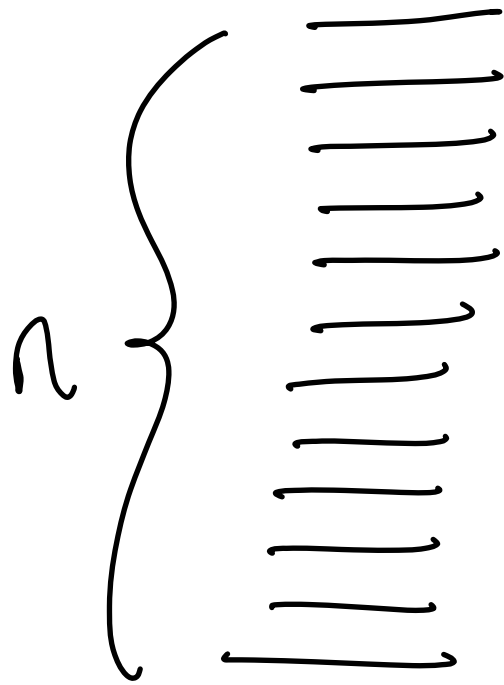


$$X \in \mathcal{P} \iff g(X) = 0$$

Proof

By "reverse" induction on topological number: Base Case $n = |X|$





n : take any even #
but not whole
pile

n odd : take whole pile

n	0	1	2	3	4	5	6	7	8	9
$g(n)$	0	1	0	2	1	3	2	4	3	

Thm

$$g(2k) = k-1$$

$$g(2k-1) = k$$

Proof by induction on k .

Base case are done on previous slide.