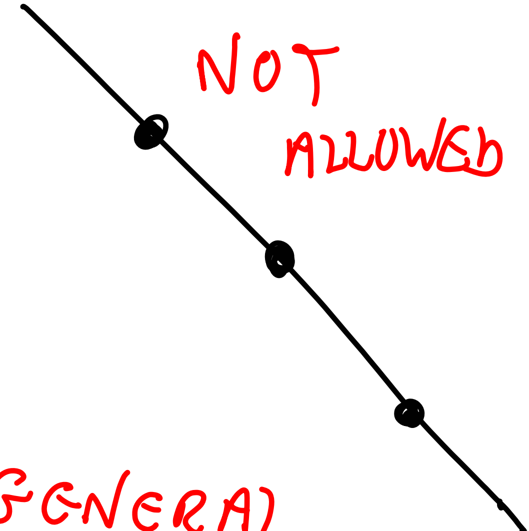
11\02\2007

$X \subseteq \mathbb{R}^2$

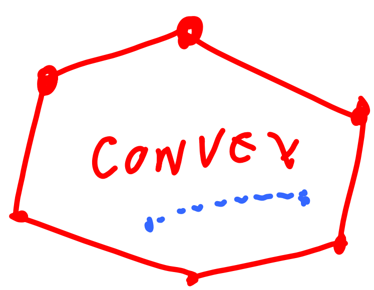


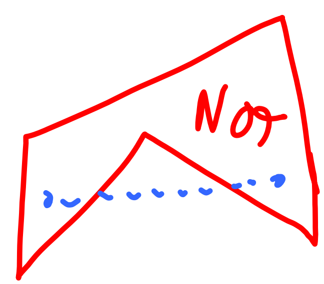
GENERAL POSITION

Claim: If $n \geq N(k,k;3) \Rightarrow \exists\, |Y|$

(i) $|Y| = k$

(ii) Y are vertices

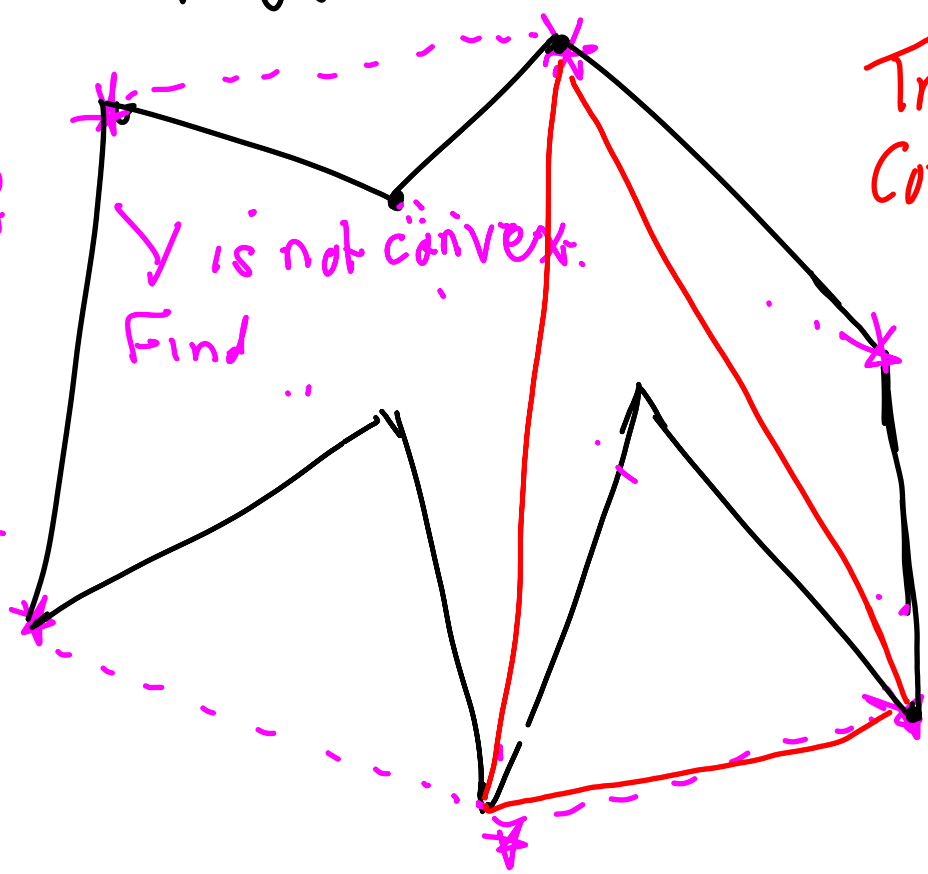




If every 4 subset of $Y \subseteq X$ forms convex quadrilateral, then Y forms convex polygon.

vertices of * = convex hull.

Y is convex iff Y = convex hull.



Triangulate convex hull.

*Y

*Y

*Y

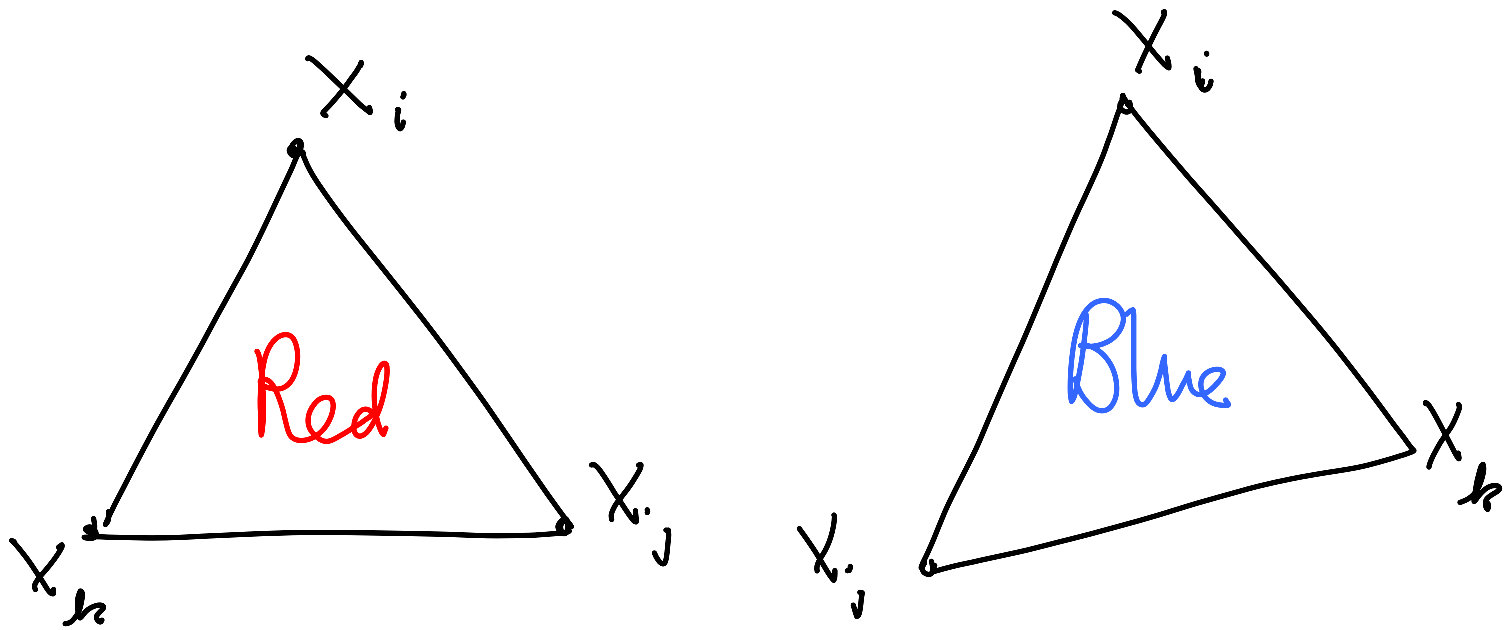
*Y

If $|X| = n \geq N(k, k; 3)$

$X = \{x_1, x_2, \ldots, x_n\}$

2-color all triples from X.

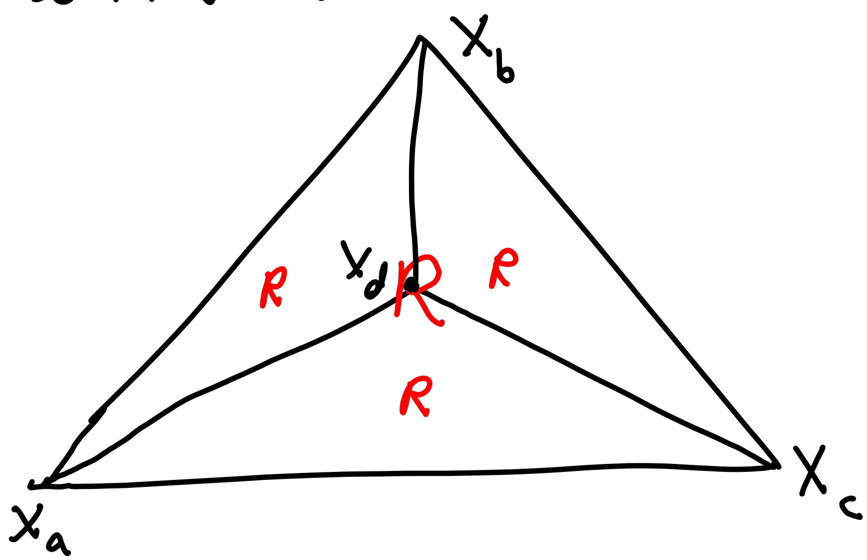
then $\exists$ convex Y, $|Y| = k$.



Take 3 points x_i, x_j, x_k $\quad i < j < k$

$\exists$ a k-set with all triples the same color:

We have to show



All 4 triples have same color.

Assume $a < b, c$

$\Downarrow$

(i) $a < b < c$

$d < a < b$

or

(ii) $a < b < d$

$a < d < c$

$b < c < d$

Partially Ordered Sets: Dilworth's Theorem

Poset: $(P, \leq)$

set — P

binary relation — $\leq$

(i) Reflexive: $a \leq a$.

(ii) Transitive: $a \leq b$ & $b \leq c \Rightarrow a \leq c$

(iii) Anti-symmetric: $a \leq b$ & $b \leq a \Rightarrow a = b$.

(i) $P = \mathbb{R}$

$\leq \; : \; \leq$

(ii) $P = \{1, 2, \dots\}$

$a \leq b$ iff a divides b

(iii) $P = \{A_1, A_2, \dots, A_m\}$

$\leq \; : \; \subseteq$

a, b are comparable if $a \leq b$ or $b \leq a$

Otherwise they are incomparable

$a < b \equiv a \leq b$ and $a \neq b$

Chain: $a_1 < a_2 < \cdots < a_s$

Anti-chain: $a_1, a_2, \ldots, a_s$

all pair-wise incomparable.

[(iii) Sperner family]

P is finite:

(I) m_1 = max. size of a chain

(II) m_2 = max size of anti-chain

(III) $C_1, C_2, \ldots, C_m$ are chains, $C_1 \cup \ldots C_m = P$

cover

minimise

(IV) $A_1, A_2, \ldots, A_n$ are anti-chain $A_1 \cup \ldots A_m = P$