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More general theorem:

(I) s colors

(II) Complete graph $= \{ \binom{n}{2}$ 2-sets $\}$

Complete $\leftarrow K_{n,r}$
r-graph $= \{ \binom{n}{r}$ r-sets $\}$

Ramsey: Let n be large enough
Color $K_{n,r}$ with s colors: Then $\exists$
"large" set $A \subseteq [n]$ s.t. $\binom{A}{r}$ is mono-colored

$n=5, \quad r=3$

$K_{5,3}$:

	Coloring 1	Coloring 2
$\{1, 2, 3\}$	R	B
$\{1, 2, 4\}$	B	R
$\{1, 2, 5\}$	R	B
$\{1, 3, 4\}$	B	B
$\{1, 3, 5\}$	R	R
$\{1, 4, 5\}$	R	R
$\{2, 3, 4\}$	B	R
$\{2, 3, 5\}$	R	B
$\{2, 4, 5\}$	R	R
$\{3, 4, 5\}$	R	B

2^{10} colorings

$s = 2$.

Do it by induction:

First on r: Assume $N(p,q;r-1)$ exists for all p,q. [We have shown this for $r = 1, 2$.]

Second on $p+q$: Assume $N(p',q';r)$ exists if $p'+q' < p+q$

Now we must show $N(p,q;r)$ exists.

$N(r,r;r) = r$ is base case.
$p+q \le 2r$ is done.

n is large

$\binom{[n]}{r}$ σ

$S+n$

$\binom{[n-1]}{r-1}$ τ

S

"Big enough"

$n-1 \geq N(N(..), N(..); r-1)$

Given coloring σ.

$|A| \geq N(p-1, q; r)$

Consider how σ colors A.

(i) $\exists$ blue Q of size q. ✓

(ii) $\exists$ red P' of size $p-1$.

$P = P' + n$

all red

$\tau(S) = \sigma(S+n)$

$\Longrightarrow$ "large" monocolored $\binom{A}{r-1}$ under τ.

Red $A \subseteq [n-1]$

Blue $B \subseteq [n-1]$ $|B| \geq N(p, q-1; r)$

4 colors: Light Red, Dark Red, Light Blue, Dark Blue

$\binom{[n]}{r}$

4-color

Apply Ramsey 2 $\Rightarrow$ large Red

or

large Blue

large Light Blue

or large Dark Blue

Schur's Theorem

$$r_k = N(\underbrace{3,3,3,3,\dots 3}_{k \text{ colors}}; 2)$$

Theorem

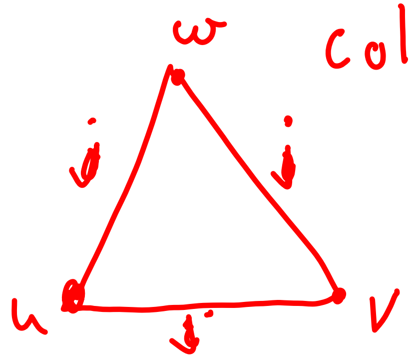
If $[r_k]$ is partitioned into $S_1, S_2, \dots, S_k$

then $\exists\, i$ & $\exists\, x,y,z \in S_i$ s.t. $x+y=z$.

Define coloring of K_n, $n \geq r_k$

(u,v): If $|u-v| \in S_i$

color it i.



$u < v < w$

$x = v-u \in S_i$

$y = w-v \in S_i$

$z = w-u \in S_i$