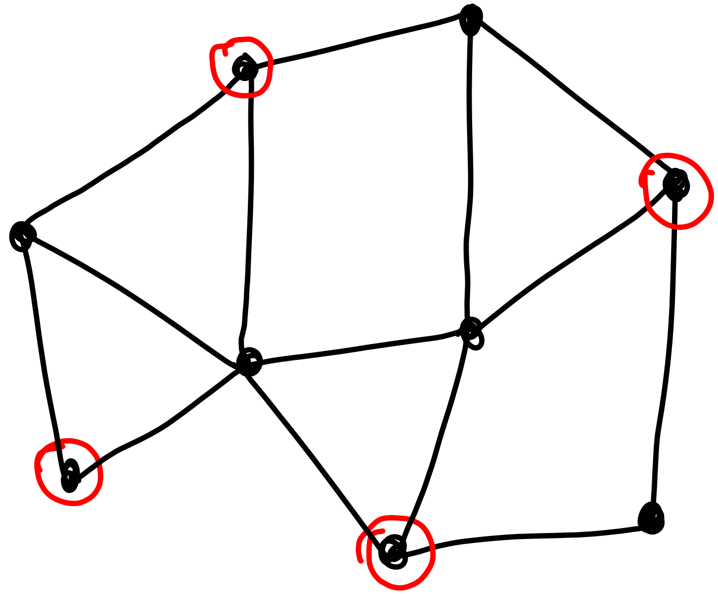
10/15/2007

$G = (V, E)$

Independent Set of vertices:




Indep.

S is independent if $\nexists\ u, v \in S$ s.t. $uv \in E$

$\alpha(G)$ = max. size of indep. set in G.

Theorem [Turán]

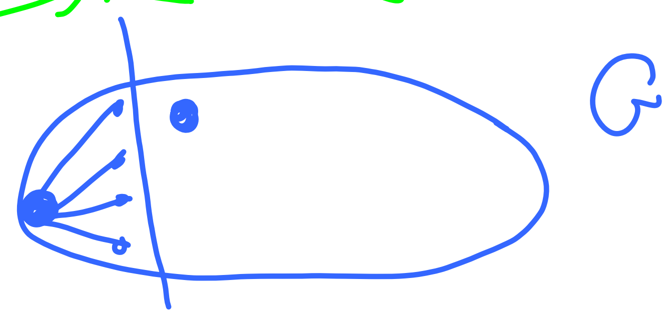
If $|V| = n$, $|E| = m$ then

$$\alpha(G) \geq \frac{n}{\frac{2m}{n} + 1}$$

average degree

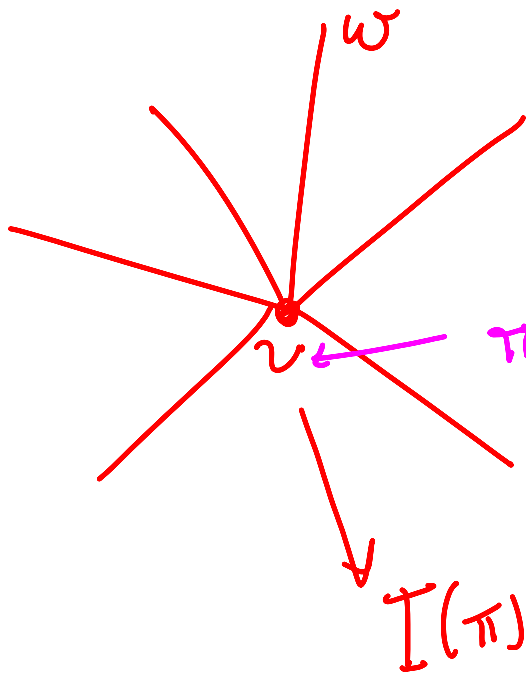
Trivial Bound: $\alpha(G) \geq \dfrac{n}{\Delta + 1}$

max. degree



Let $\pi(1), \pi(2), \ldots, \pi(n)$ be an arbitrary permutation of V.

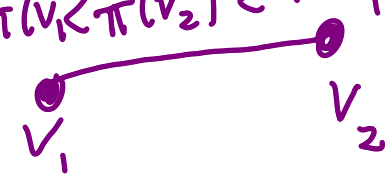
$$I(\pi) = \{ v : \pi(w) > \pi(v), \text{ for all nbrs } w \text{ of } v \}$$



$\pi(v) < \pi(w)$, $\forall w$ adjacent to v.

$I(\pi)$ is independent ??

$\pi(v_1 < \pi(v_2) < \pi(v_1)$



If π is a random permutation then

$$E(I) = \sum_{v \in V} \frac{1}{d(v)+1}$$

<u>Proof</u>

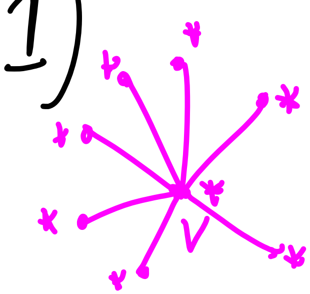
$$\delta(v) = \begin{cases} 1 & v \in I \\ 0 & v \notin I \end{cases} \qquad \text{indicator for } I$$

$Pr(\pi(v) \text{ is smallest of } *)$

$$|I| = \sum_{v \in V} \delta(v)$$

$$E(|I|) = \sum_{v \in V} E(\delta(v)) = \sum_{v \in V} Pr(\delta(v) = 1)$$

$$= \sum_{v \in V} \frac{1}{d(v)+1}.$$



Therefore

$$\alpha(G) \geq \sum_{v \in V} \frac{1}{d(v)+1}$$

$$\overset{?}{\geq}$$

Arithmetic Mean : $\dfrac{x_1 + x_2 + \cdots + x_k}{k}$

$\mathbin{\rotatebox[origin=c]{-90}{$\geq$}}$

Geometric Mean : $(\underset{>0}{x_1}\,\underset{>0}{x_2} \cdots \underset{>0}{x_k})^{1/k}$

$\mathbin{\rotatebox[origin=c]{-90}{$\geq$}}$

Harmonic Mean : $\left[\dfrac{1}{k}\left(\dfrac{1}{x_1} + \dfrac{1}{x_2} + \cdots + \dfrac{1}{x_k}\right)\right]^{-1}$

$$\left[\frac{1}{k}\left(\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_k}\right)\right]^{-1} \leq \frac{x_1+\cdots+x_k}{k}$$

$$\frac{1}{k}\left(\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{k}\right) \geq \frac{k}{x_1+\cdots+x_k}$$

$$\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_k} \geq \frac{k^2}{x_1+\cdots+x_k} \qquad (*)$$

$k = n$

$x_v = d(v)+1.$

$$\sum_{v \in V} \frac{1}{d(v)+1} \geq \frac{n^2}{\sum_v [d(v)+1]} \leftarrow 2m+n.$$

$$\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_k} \geq \frac{k^2}{x_1 + \cdots + x_k} \quad (*)$$

$$\underbrace{(x_1 + \cdots + x_k)\left(\frac{1}{x_1} + \cdots + \frac{1}{x_k}\right) = k + \sum_{1 \leq i < j \leq k} \underbrace{\left[\frac{x_i}{x_j} + \frac{x_j}{x_i}\right]}_{\geq 2}}_{\geq k + 2\binom{k}{2} = k^2.} \quad \square$$

$$\alpha + \frac{1}{\alpha} \geq 2$$

$$\Updownarrow$$

$$\underbrace{\alpha^2 - 2\alpha + 1}_{(\alpha - 1)^2} \geq 0$$

Parallel Searching For Maximum

Parallel Machine:

n processors

In a "round" of computation, each computer independently.

$x_1, x_2, \ldots, x_n$ are numbers.

In any one round, a machine can compare 2 numbers and report the largest.