21-301 Combinatorics
Homework 9
Due: Wednesday, November 29

1. Prove that if we 2-color the edges of K_6 then there are least *two* mono-chromatic triangles.

2. Prove that if $n \geq R(2k, 2k)$ and if we 2-color the edges of $K_{n,n}$ then there is a mono-chromatic copy of $K_{k,k}$.

3. Let $I_1, I_2, \ldots, I_{mn+1}$ be closed intervals on the real line i.e. $I_j = [a_j, b_j]$ where $a_j \leq b_j$ for $1 \leq j \leq mn+1$. Use Dilworth's theorem to show that either (i) there are $m+1$ intervals that are pair-wise disjoint or (ii) there are $n+1$ intervals with a non-empty intersection.