21-301 Combinatorics
Homework 8
Due: Wednesday, November 8

1. Let $s \geq 1$ be fixed. Let $\mathcal{A}$ be a family of subsets of $[n]$ such that **there do not exist** distinct $A_1, A_2, \ldots, A_{s+1}$ such that $A_1 \subseteq A_2 \subseteq \cdots \subseteq A_{s+1}$. Show that
$$\sum_{A \in \mathcal{A}} \frac{1}{\binom{n}{|A|}} \leq s.$$

2. Suppose that $2^m > mn - \binom{m}{2} + 1$ and S, $|S| = m$ is a subset of $[n]$. Show that there exist distinct $A, B \subseteq S$ such that (i) $A \cap B = \emptyset$ and (ii) $\sum_{a \in A} a = \sum_{b \in B} b$.

3. Given any sequence of n integers, positive or negative, not necessarily all different, show that some consecutive subsequence has the property that the sum of the members of the subsequence is a multiple of n.