

21-301 Combinatorics

Homework 5

Due: Wednesday, October 11

1. A box has m drawers; Drawer i contains g_i gold coins and s_i silver coins, for $i = 1, 2, \dots, m$. Assume that one drawer is selected randomly and that a randomly selected coin from that drawer turns out to be gold. What is the probability that the chosen drawer is drawer 1?
2. A bag contains n balls, each of a different color. In a round, a person picks a random ball from the bag, makes a note of its color and then puts it back. What is the expected number of rounds required for the person to have pulled out a ball of each color at least once?
3. A particle sits at the left hand end of a line $0 - 1 - 2 - \dots - L$. When at 0 it moves to 1. When at $i \in [1, L - 1]$ it makes a move to $i - 1$ with probability $1/4$ and a move to $i + 1$ with probability $3/4$. When at L it stops.

Let E_k denote the expected number of visits to 0 if we started the walk at k .

- (a) Find a set of equations satisfied by the E_k .
- (b) Given that $E_k = \frac{A}{3^k} + B$ is a solution to your equations for some A, B , determine A, B and hence find E_0 .