21-301 Combinatorics
Homework 2
Due: Wednesday, September 13

1. Show that
$$\sum_{k=1}^{n} \binom{n}{k} \frac{k}{n^k} = \left(1 + \frac{1}{n}\right)^{n-1}.$$
[Hint: Differentiate the expression in the binomial theorem and then put in a suitable value for x.]

2. Show that for a fixed k,
$$\sum_{\ell=0}^{n-k} \binom{n}{k, \ell, n-k-\ell} 2^\ell = 3^{n-k} \binom{n}{k}.$$
[Hint: Expand $(1+x+y)^n$ using the multinomial theorem. Then put $y = 2$ and extract the coefficient of x^k in what remains.]

3. Find an expression for the size of the set
$$\{(x_1, x_2 \ldots, x_m)\} \in Z^m : \ x_1 + x_2 + \cdots + x_m = n \ and \ 0 \le x_j \le a \ for \ j = 1, 2, \ldots, m\}.$$
[You should use Inclusion-Exclusion and expect to have your answer as a sum.]