21-301 Combinatorics
Homework 10
Due: Wednesday, December 6

1. Consider the following game: There is a pile of n chips. A move consists of removing any *proper* factor of n chips from the pile. (For the purposes of this question, a proper factor of n, is any factor, including 1, that is strictly less than n). The player to leave a pile with one chip wins. Determine the N and P positions and a winning strategy from an N position.

2. Consider the following game: There is a single pile of n chips. A move consists of removing (i) any *even* number of chips provided it is not the whole pile, or (ii) the whole pile, but only if it has 2 (mod 3) chips. The terminal positions are zero and one. Determine the Sprague-Grundy numbers of each pile size.
(Compute the first 15 numbers and see if you can see see a pattern.)

3. Consider the following multi-pile game. A move consists of either (i) removing one, two or three chips from any pile, or (ii) splitting a pile of size $n \geq 2$ into two piles of sizes 1 and $n - 1$. Determine the Sparague-Grundy numbers for a game that starts with a single pile of size n.
(Compute the first 15 numbers and see if you can see see a pattern.)

Suppose that the current position consists of three piles of sizes 3,5 and 7. Show that this is an N-position and find all of the winning moves.