

21-301 Combinatorics
Homework 1
Due: Wednesday, September 6

1. How many integral solutions of

$$x_1 + x_2 + x_3 + x_4 = 40$$

satisfy $x_1 \geq 2$, $x_2 \geq 0$, $x_3 \geq -5$ and $x_4 \geq 5$?

2. Prove the following equality using a *combinatorial* argument

$$\sum_{i=2}^n \binom{i}{2} \binom{n}{i} = \binom{n}{2} 2^{n-2}.$$

3. A sequence $a_1 a_2 \cdots a_n$ where $a_i \in [m]$ is *increasing* if $a_{i+1} \geq a_i$ for $1 \leq i < n$. Show that the number of such increasing sequences is $\binom{m+n-1}{n}$.