11\29\06

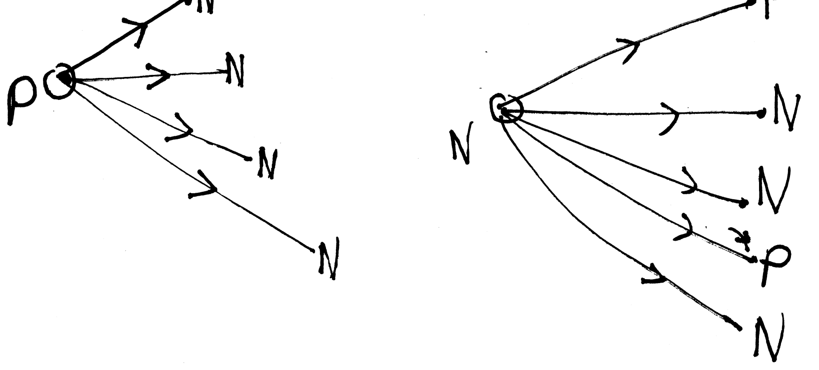
Games as defined must terminate: the topological number of the position strictly increases on each move.

Two sorts of position:

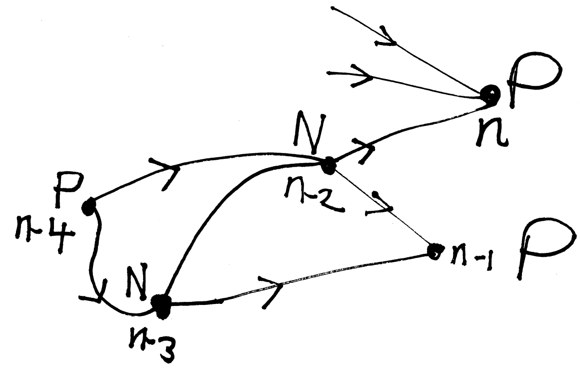
P - position, win for previous player

N - postion, win for next player

Labelling algorithm to find N & P positions



Label in <u>decreasing</u> topological number.

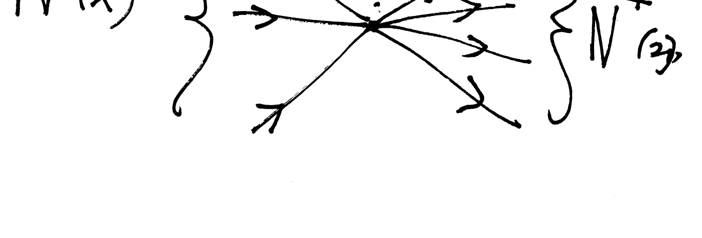


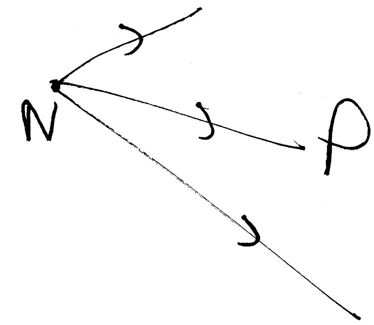
Partition into N & P s.t.

**

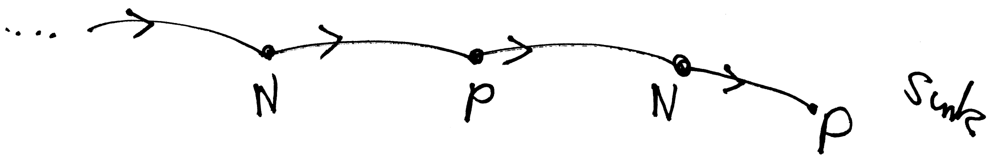
$$\boxed{x \in N \text{ iff } N^{+}(x) \cap P \neq \emptyset}$$

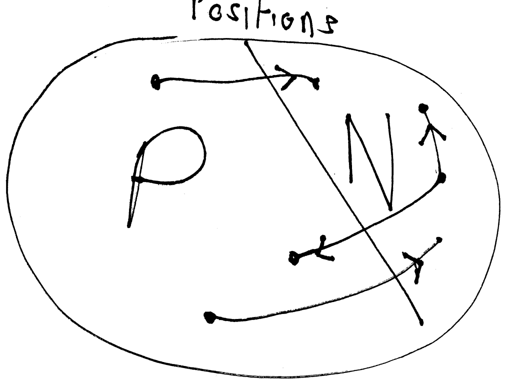
out-nbrs of x





Game should go





Only one split into N & P satisfying $\boxed{**}$
Induction on top. numbering

Sums of games

Suppose we n games, $D_i = (X_i, A_i)$

Sum of the games is played on

$$X_1 \times X_2 \times \dots \times X_n$$

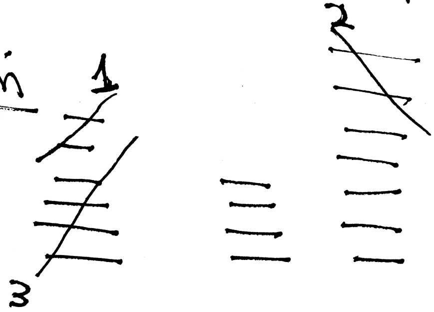
A position is a vector $(x_1, x_2, \dots, x_n)$, $x_i \in X_i$

In other words, n positions one in each component game.

One move: player chooses i and moves in game i.

Game ends when all positions are sinks.

Nim



Several piles:
Game i = one pile game on i'th pile

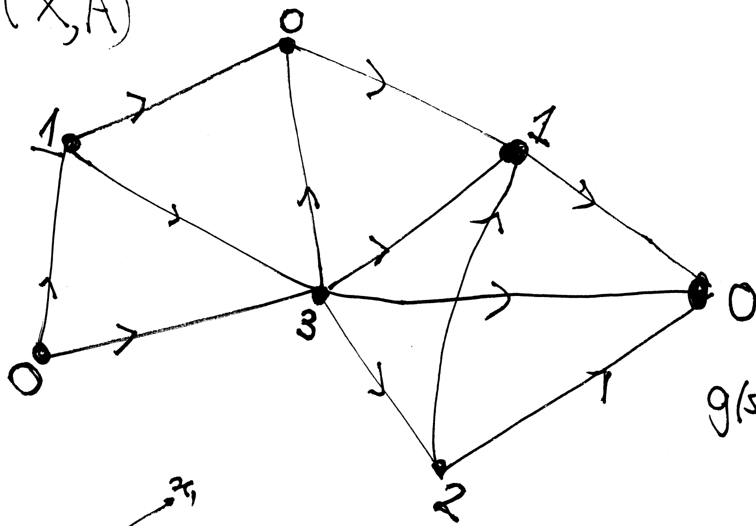
Knowing (N_i, P_i) $i=1,2,\dots,n$ is not enough to work out how to play.

Need more information:

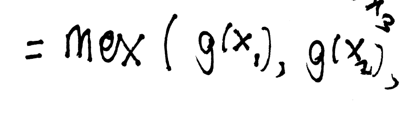
Sprague-Grundy Numbering. g

One game

$D=(X,A)$

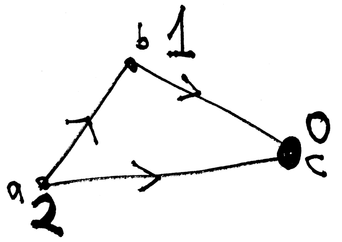


$g(\text{sink}) = 0$

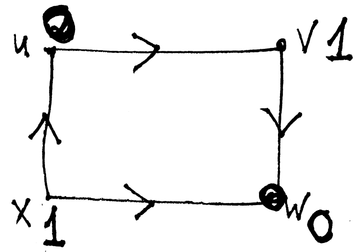


$g(a) = \text{mex}\,(g(x_1), g(x_2), g(x_3))$

mex ≡ minimum excluded.

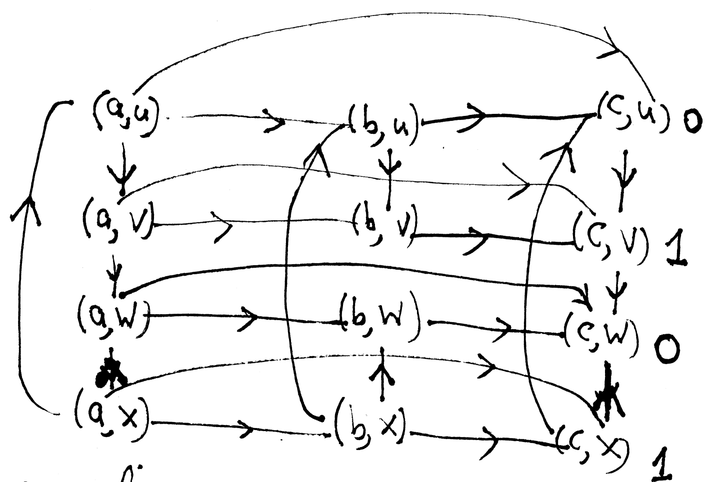


Game 1



Game 2

Game1 + Game 2

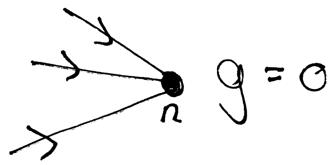


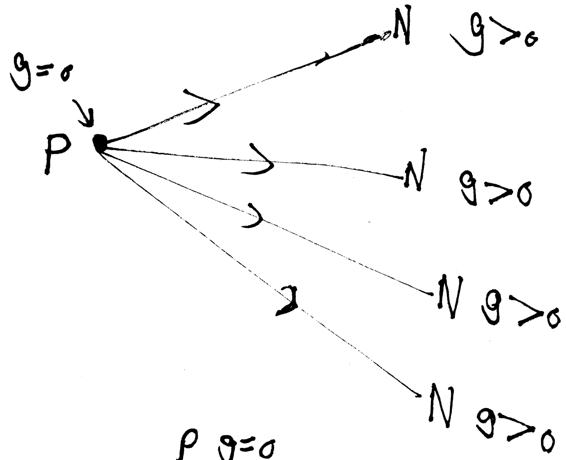
One property:

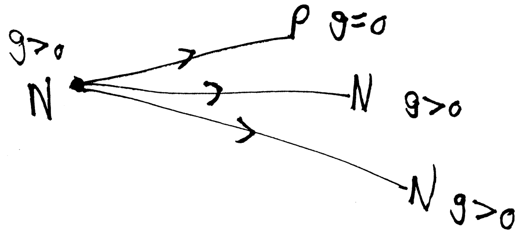
$$x \in P \leftrightarrow g(x) = 0.$$

$$x \in P \leftrightarrow g(x) = 0.$$

Backwards induction on topological number.







What is the use of these numbers:

(i) Knowing g(position) tells how to win.

Game = Game1 + Game2 + ...

then

$$g(\text{Game}) = g_1(\text{Game }1) \oplus g_2(\text{Game }2) \oplus \ldots$$

$\oplus$ = bit-wise addition
no carry.

$5 \oplus 7$

$$\begin{array}{rr} 101 & 5 \\ 111 & 7 \; \oplus \\ \hline 010 & 2 \end{array}$$