11/10/06

Existence of $N(q_1, q_2, \ldots, q_s; r)$

$S=2$, <u>done</u>.

<u>Example</u>

$$N(k, k, k, k; r)$$

$$\leq \underbrace{N\Big(\underset{Q_1}{\underset{\shortparallel}{N(k,k;r)}}, \underset{Q_2}{\underset{\shortparallel}{N(k,k;r)}}; r\Big)}_{\text{exists because only uses 2 colors}}$$

4 colors: $\underbrace{\text{light red, dark red}}_{\text{Red}}$, $\underbrace{\text{light blue, dark blue}}_{\text{Blue}}$

Either $\exists Q_1$ s.t. all of whose r-sets are Red (I)

$\exists Q_2$ s.t. all of whose r-sets are Blue (II)

Assume (1) hold.

We have a 2-coloring of the r-sets of the Q_1-set.

lightred or dark red

$Q_1 = N(k, k_2; r)$ and so $\exists$ a required k-set.

Schur's Theorem

$r_k = N(\underbrace{3,3,\cdots,3}_{k \text{ of them}};\ 2)$

[Coloring K_N's edge and looking for a mono-chromatic Δ]

Theorem

If you partition $\{1,2,\cdots,r_k\}$ into k-sets $S_1, S_2, \cdots, S_k$ then $\exists\ i, a, b, c$ such that $a, b, c \in S_i$ and $a+b=c$. [a, b donot have to be distinct]

Example

$k=2$

$r_k=6$

$1, 2, 3, 4, 5, 6$

$S_1 = 3, 4, 6$

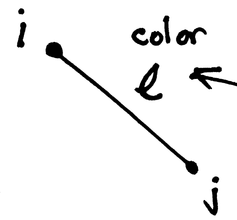
$S_2 = 1, 2, 5$

✓

Proof

Given $S_1, S_2, \dots, S_k$ which partition $\{1,2,\dots,\overset{r_k}{\underset{n}{\ }}\}$

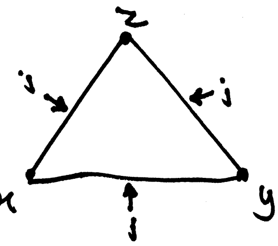
Use the partition to color the edge of K_n.



$|i-j| \in S_\ell$

I get a k-coloring of K_n

$\exists$ a mono-chromatic Δ.



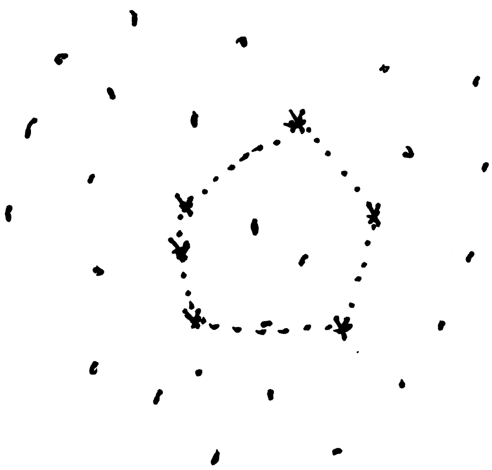
Assume $x < y < z$

$u = y - x \in S_j$

$v = z - y \in S_j$

$w = z - x \in S_j$

$u + v = w$



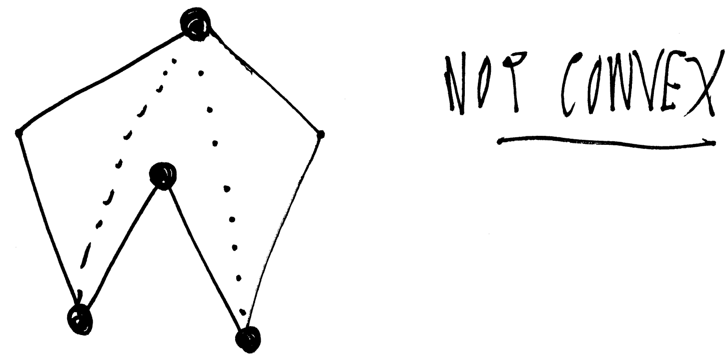
n points. $n \geq N(k,k;3)$

$\Rightarrow$ $\exists$ a set of k points forming a convex polygon.

No 3 points are in a line.

Observation: if every 4-subset of a set of points X form a convex quadrilateral, then X define a convex polygon.

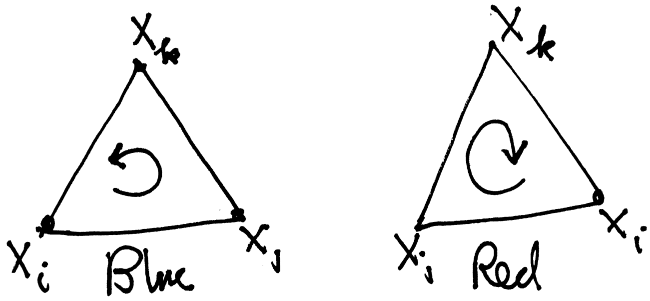
Why?



So I need to find a set of k points all of whose 4-subsets form a convex quadrilateral.

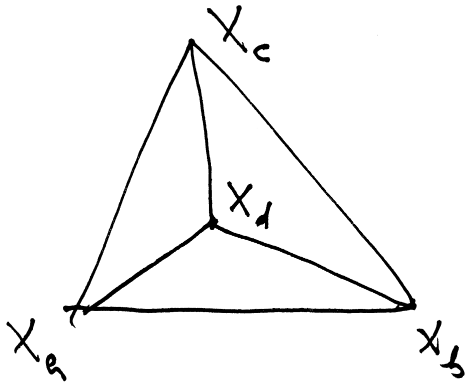
Suppose points are $X_1, X_2, \ldots, X_n$

I color triangle $\{i, j, k\}$ where $i < j < k$



Since $n \geq N(k,k;3)$, $\exists$ a k-set with all Δ's the same color.

$\Rightarrow \nexists$



All triangle are same color.