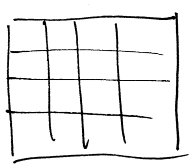
Let $m = \left\lfloor \sqrt{\frac{n-1}{2}} \right\rfloor$ split the square into m^2 smaller ones.

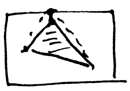
we have $m^2 \le \left(\sqrt{\frac{n-1}{2}}\right)^2 = \frac{n-1}{2} < \frac{n}{2}$



By the pigeonhole principle

look at $\frac{n}{m^2} > 2$ so by PH

there is a square with $\left\lceil \frac{n}{m^2} \right\rceil \ge 3$

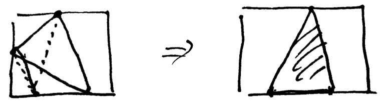


$\frac{1}{m}$

$\frac{1}{m}$

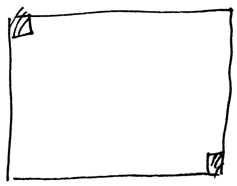
area is $\le \frac{1}{2}$ area of square

$= \frac{1}{2} \cdot \frac{1}{m^2}$



Ok

Chess board



Can I tile this with dominoes?

32 white squares

30 black

HW Problems

Last month (28 days) I did 40 HW* problems. (at least one a day). Can you find consecutive days s.t. I solved a total of 15 ~~day~~ problems.

Soln

Let x_j = # of problems solved (in total) at the end of day j.

All the x_j's are distinct. Does there exist j,k s.t. $x_j + 15 = x_k$

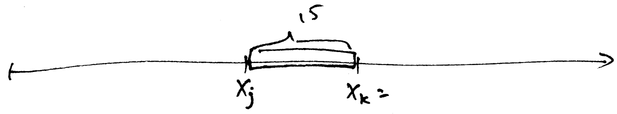
what are possible values of x_j : $\in \{1, \dots, 40\}$

$$x_j + 15 \in \{16, \dots, 55\}$$

So both x_j and $x_j + 15 \in \{1, \dots, 55\}$

The x_j's and (x_j+15)'s are 56 in total.

So there has to be a number $a \in \{1, \dots, 55\}$ s.t. two x or $x+15$ map to the same thing. There has to be x_j+15 and x_k that take on t[he] same value. (since the x_j's are distinct)



$X_\cdot : [28] \to [55]$

$X_\cdot + 15 : [28] \to [55]$

create $Y_\cdot : [56] \to [55]$

$$Y_j = \begin{cases} X_j & \text{if } j \in [28] \\ X_{j-28} + 15 & \text{if } j \in \{29, \dots, 56\} \end{cases}$$

So by PH there has to be a $a \in [55]$

$y_j = a = y_k$ $\quad j \neq k$.

__Acquaintances__

$N > 1$ people in a room, show that two people know the same # of people.

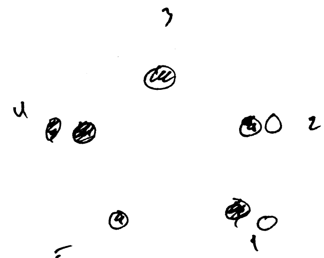
__Solution__

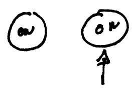
Throw out people with 0 friends.

We can assume that # of friends $\in \{1, \dots, N-2\}$. $\subseteq \{1, \dots, N-1\}$

So by PH ....

N lightbulbs in a circle, initially all are on. At time t we look at lightbulb t (mod N), if the lightbulb is on I'll modify the state of t+1 (mod N)

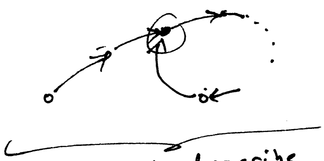




Will we ever get to the state where all lightbulbs are on?

Look at the state space: $N2^N < \infty$

This process is time reversible.



does not describe our process.