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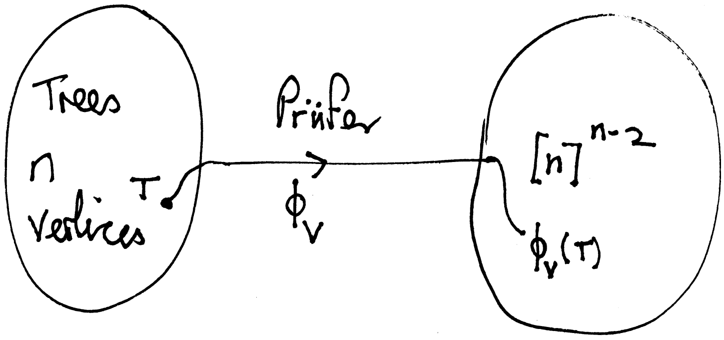
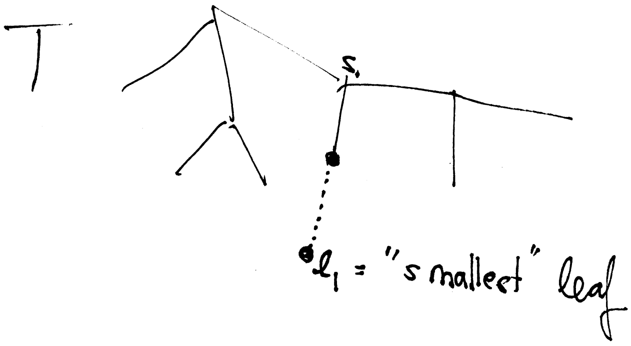




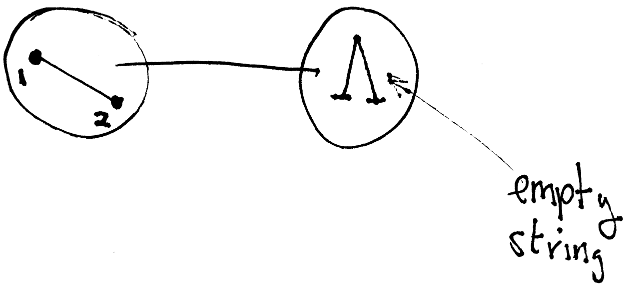
s_1 . . . .

$\xleftarrow{\quad ?? \quad}$

We show ϕ_V has an inverse.

Prove existence of Φ_V^{-1} by induction on $|V|$.

$|V| = 2$.

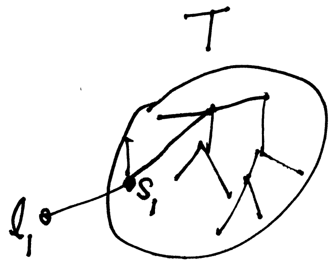


Sequence $s_1 s_2 \dots s_{n-2}$

$\downarrow$

Tree

(i) Find l_1 = smallest value not mentioned in $s_1, s_3, \dots, s_{n-2}$



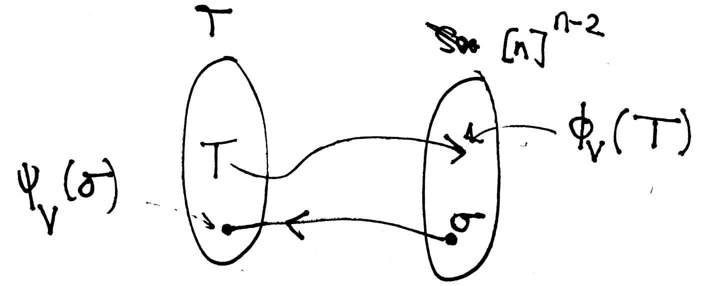
(ii) $V \longrightarrow V / \{l_1\}$

Build $T_1 = T - (l_1, s_1)$ from $s_2, s_3, \dots s_{n-2}$ [Use induction (recursion)]

$$\underbrace{\psi_V \cancel{\phi_V^{-1}}(s_1 s_2 \dots s_{n-2})}_{\text{tree } T} = \text{edge}(l_1, s_1) + \text{tree } T_1,$$

$$T_1 = \phi_{V_1}^{-1}(s_2 s_3 \dots s_{n-2})$$

where $V_1 = V \setminus \{l_1\}$.



<u>Claim</u> $\psi_V = \phi_V^{-1}$

$$\phi_V(\psi_V(s_1 s_2 \dots s_{n-2}))$$

$$= \phi_V((l_1, s_1) + T_1)$$

$$= s_1 \phi_{V_1}(T_1)$$

$$= s_1 s_2 \dots s_{n-2}.$$

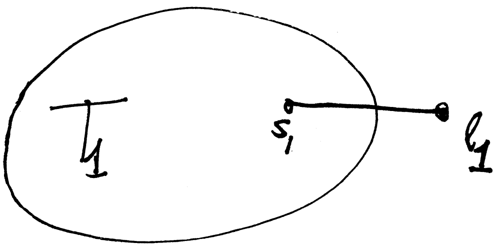
$T \longrightarrow \phi_v(T)$

$v \in T$ appears exactly $d_T(v) - 1$ time.

[Leaves do not appear in the sequence]

<u>Proof by induction on $|V|$</u>

$|V| = 2$ ✓



l_1 appears 0 times

s_1 appear $1 + \underbrace{\underbrace{\#\text{times in } \phi_{v_1}(T_1)}_{} = \underbrace{\deg_{T_1}(s_1) - 1}_{}}_{\substack{\deg_{T_1}(s_1) \\ = \deg_T(s_1) - 1}} + 1$

Every other vertex x has
$\deg_{T_1}(x) = \deg_T(x)$.

$\#$ trees with degree sequence $d_1, d_2, \cdots, d_n$

$d_1 + d_2 + \cdots + d_n = 2n-2$

$= \#$ Sequences in $[n]^{n-2}$ where i appears $d_i - 1$ times, $\forall i$

$$= \binom{n-2}{d_1-1, d_2-1, \ldots, d_n-1}$$

Cayley's formula $= \#$ spanning trees of K_n

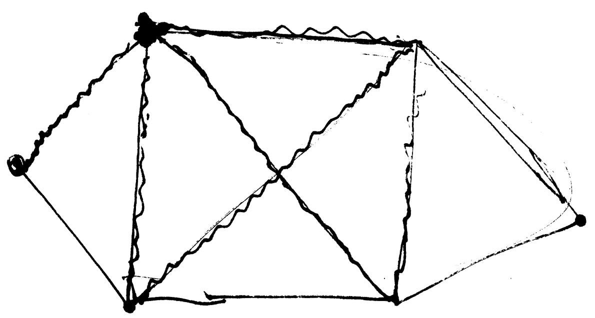
Arbitrary connected graph G then

$\#$ spanning trees of G $=$ determinant $\left(\begin{array}{c}\text{Some } (n-1)\times(n-1) \\ \text{matrix}\end{array}\right)$

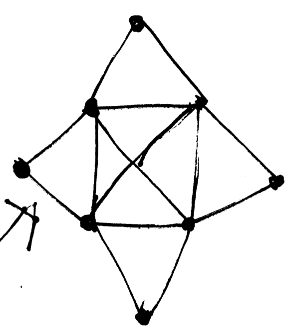
<u>Matrix Tree Theorem</u>

Eulerian Graphs

An Eulerian cycle is a closed walk through G which goes through each edge exactly once.



Q? Which graphs have Eulerian cycles?



NO

First result in Graph Theory

Bridge of Konigsberg Problem.

Thm

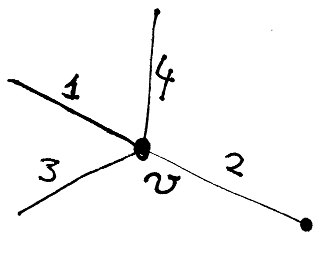
G has an Euler Cycle iff

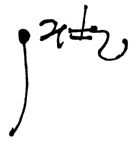
(i) G is connected

(ii) Every degree is even.

Proof

Euler Cycle $\longrightarrow$ even degrees.





Assume $\exists$ euler cycle C.

Traverse it.

We won't start at v

We start at x

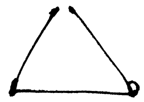
2 vertices of odd degree

Now eulerian

Euler trail

eulerian $\longleftarrow$ even & connected

By induction on $|E|$.

Base Case  or  or 

Inductive Step.

G has at least one cycle C

Remove C