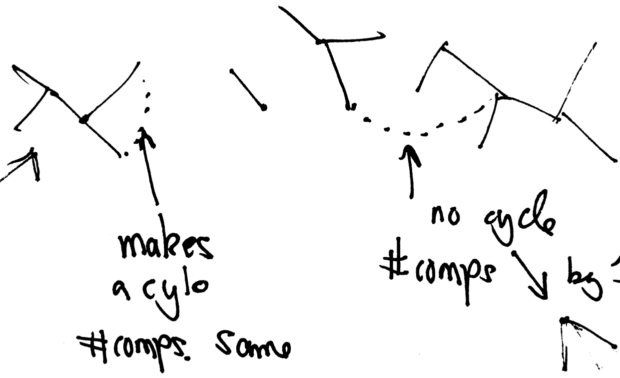
10\23\06

Trees – connected, no cycles.

Forest:



$G=(V,E)$

$|V|=n$ $\quad |E|=r$ and no cycle.

comps $= n-r$.

$E=\{e_1,e_2,\dots,e_r\}$

$G_i=\{e_1,e_2,\dots,e_i\}$
No cycle

$G_i \longrightarrow G_{i+1}$

#components goes down by 1

Started with n components

Added r edge

#components $= n-r$.

So, a forest with r edges has $n-r$ components.

OR

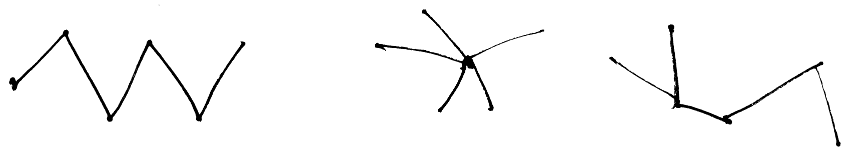
a forest with k components, has $n-k$ edges.

$\Downarrow$

A tree has $n-1$ edge

$\uparrow$

forest with one component.



$n = 6$

Trees are "smallest" connected graphs

(b) A tree has at least 2 vertices of degree 1

$$2n-2 = \sum_{v \in T} \underbrace{d_v(T)}_{\geq 1} \geq s + 2(n-s)$$

$s = \#$ vertices of degree 1

$$2n-2 \geq 2n-s$$

$$s \geq 2$$

$G = (V, E)$

Given $|V| = n$, $|E| = n-1$

Following are equivalent given

(a) G is connected

(b) G has no cycles

(c) G is a tree

(a) $\to$ (b)

G is connected. $G_0 \to G_1 \to G_2 \dots \to G_{n-1} = G$

n vertices

n-1 edges

G_0: n component

Drop = ~~0~~ or 1 (# component)

G_{n-1}: 1 component

(b) $\to$ (c)

G has n vertices

n-1 edges

no cycles

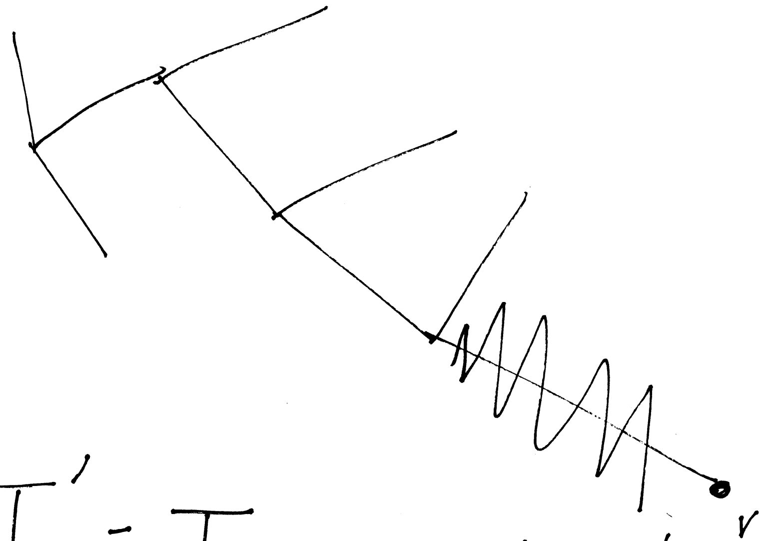
$G_0 \to G_1 \to G_2 \dots G_{n-1} = G$

G_0: n component

drop ~~0~~ or 1

Don't make a cycle

$G_{n-1} = G$: $\Rightarrow$ # components $= 1$



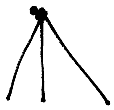
$T' = T - v$ $\deg(v) = 1$

T is a tree $\Longrightarrow$ T' is a tree. T has n vertices
$n-1$ edge

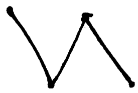
T' has $n-1$ vertices
$n-2$ edges
No cycles
$\Downarrow$
T' is a tree

#of spanning tree of K_n

$n=4$



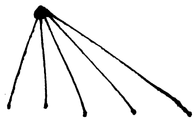
4



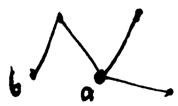
$\frac{4!}{2} = 12$

#16

$n=5$



5



$\underset{a}{5} \times \underset{b}{4} \times 3$

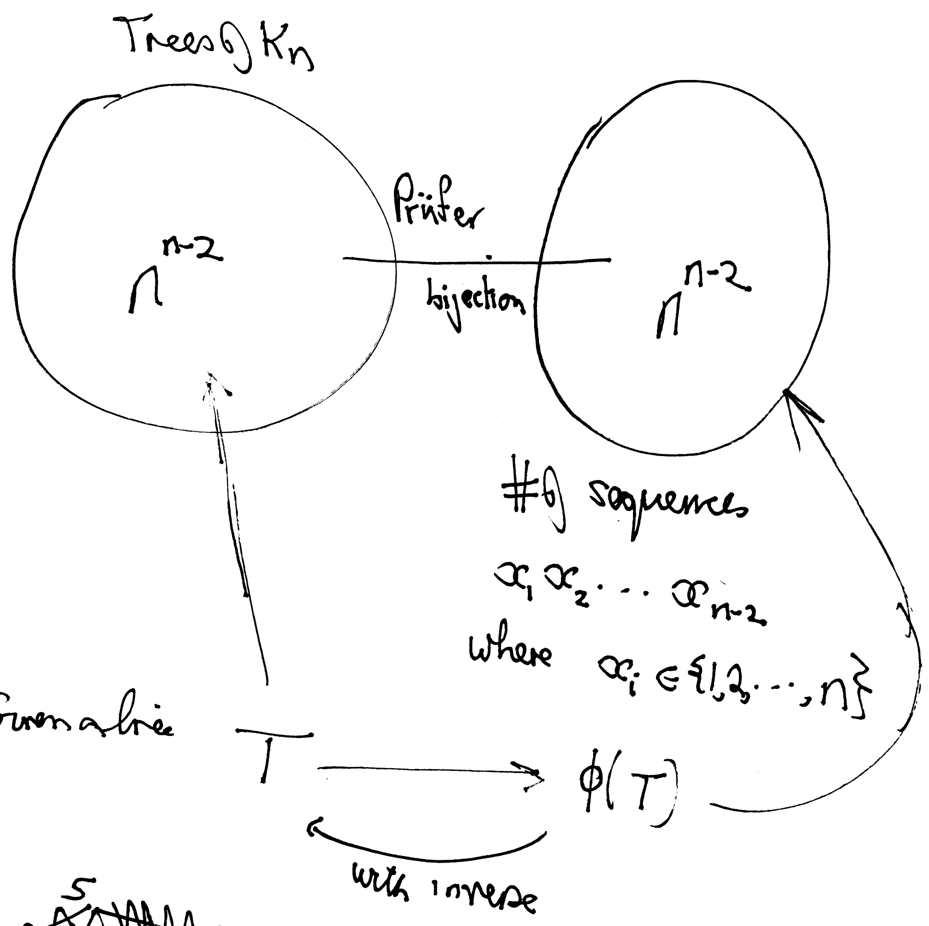
how to connect b

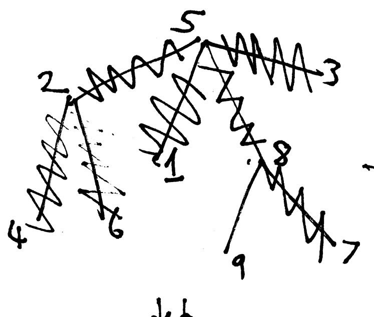
$\frac{5!}{2} = 60$

#125

16, 125, 1296 . . . . , n^{n-2}

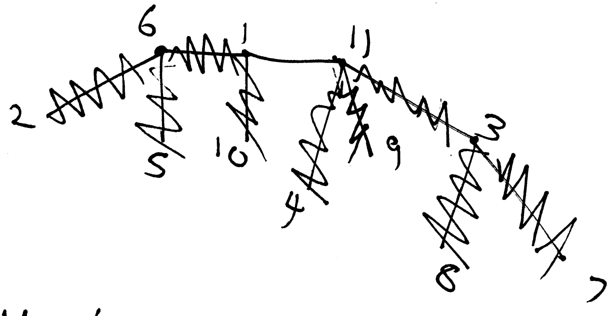
Cayley's Formula





$\downarrow \phi$

$$\underbrace{5,5,2,2,5,8,8}_{7}$$



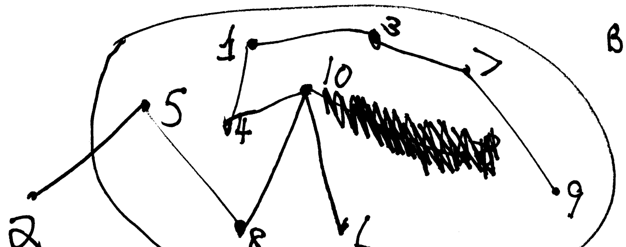
6, 11, 6, 1, 3, 3, 11, 11, 1

$$T \longrightarrow \phi(T) \longrightarrow \phi^{-1}\phi(T)$$

What is ϕ^{-1}.

n=10

5, 8, 10, 10, 7, 3, 1, 4.



Build a tree on 1,3,4,5,6,7,8,9,10 from 8,10,10,7,3,1,4

4, 10