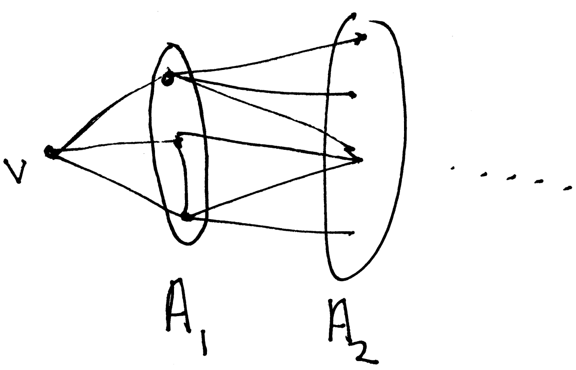
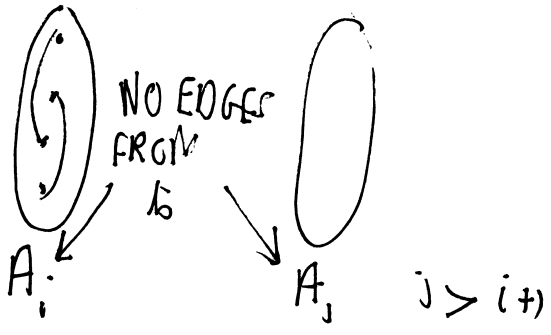
10/18/06

Breadth First Search



$A_t = \{ w : dist(v, w) = t \}$

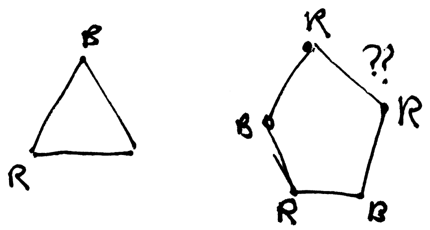
We construct the component containing v in the way



We can use BFS to try to 2-color a graph.

Theorem

What are the smallest non-2-colorable graphs.

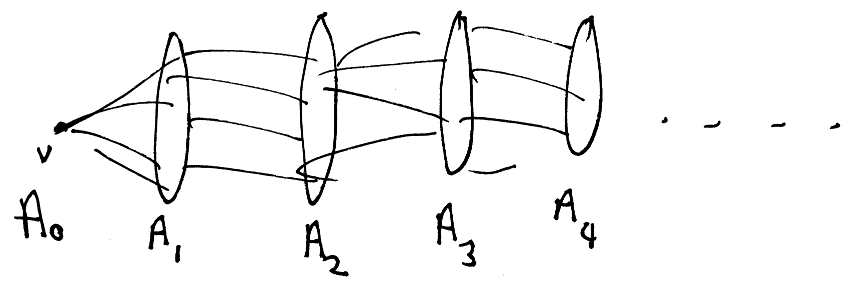


So if $G \supseteq$ an odd cycle, then G is not 2-colorable (= bipartite).

Use BFS to show that if G has no odd cycles then G is bipartite.

G has no odd cycles. It is bipartite.

Do BFS

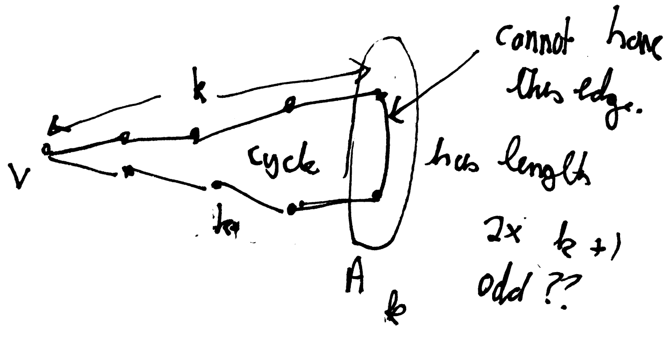


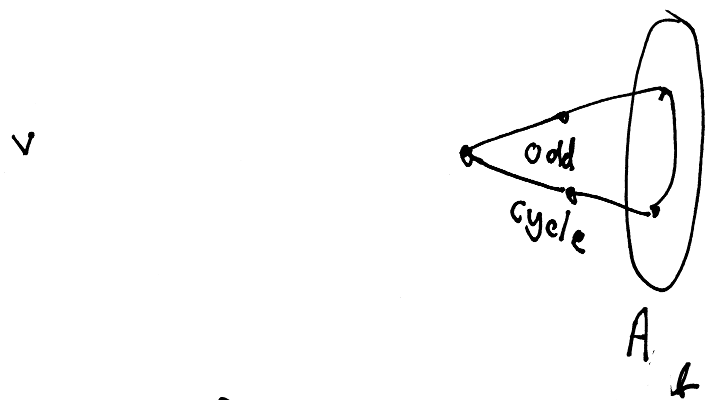
If G has no odd cycles then we have a bipartition

$(A_0 \cup A_2 \cup A_4 \cup \ldots)$ & $(A_1 \cup A_3 \cup A_5 \cup \ldots)$

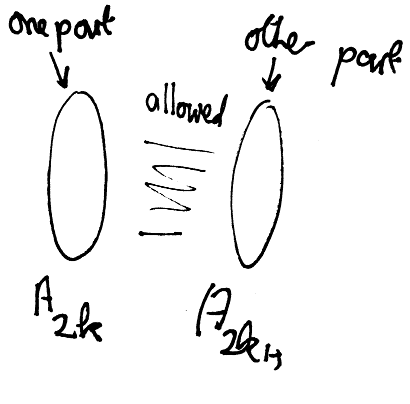
No edges inside here — No edge inside here.

A_2 NONG A_6 BECAUSE OF BFS





BFS only fails when $\exists$ an odd cycle.



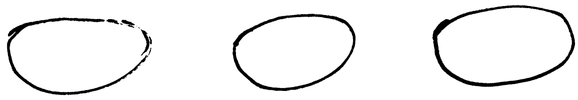
Trees

Tree is (i) Connected

(ii) No cycles.

Lemma

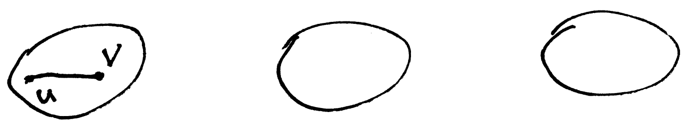
G:



Add a new edge (u,v)

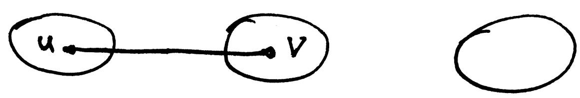
2 possibilities

(i) u,v in same component



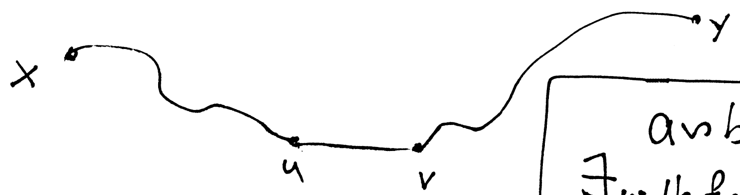
components stays same.

(ii)



u,v in different component

components goe down by 1.



$$\boxed{\begin{array}{c} a \sim b \text{ if} \\ \exists \text{ walk from } a \text{ to } b \end{array}}$$

New path:

If $u, v \in C_i \Rightarrow x, y \in C_i$ No change to relation $\sim$

$u \in C_i$, $v \in C_j$ $\Rightarrow$ $x \in C_i$, $y \in C_j$ Change to $\sim$

and for $u \in C_i$, $v \in C_j$

$G = (V,E)$ is a forest i.e. no cycles.

Every component is a tree.

Components $C_1, C_2, \ldots, C_k$

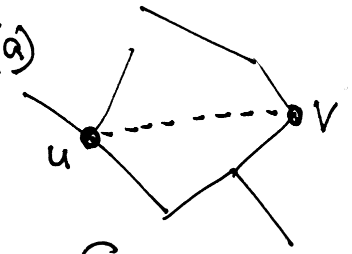
$(u,v) \notin E \qquad u \in C_i,\ v \in C_j$

(a) $i = j \Rightarrow G + e$ has a cycle

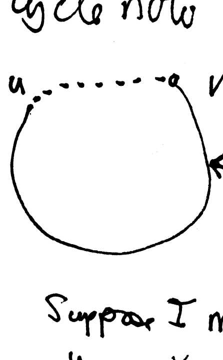
(b) $i \neq j \Rightarrow G + e$ has no cycle and one less component

(c) G has $n - k$ edges

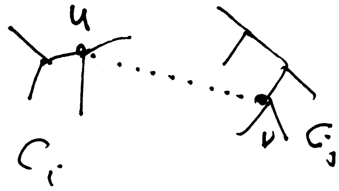
(a)



∃ cycle now



(b)



Suppose I make a cycle

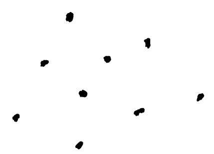
∃ u,v in same comp.

$G = C_1 + C_2 + \cdots + C_k$

$E = \{e_1, e_2, \ldots, e_\ell\}$?? $\ell = n-k$??

$G_i = (V, \{e_1, e_2, \ldots, e_i\})$

G_0



G_1

$\vdots$

G_ℓ :

C_1 C_2 C_3 .. C_k