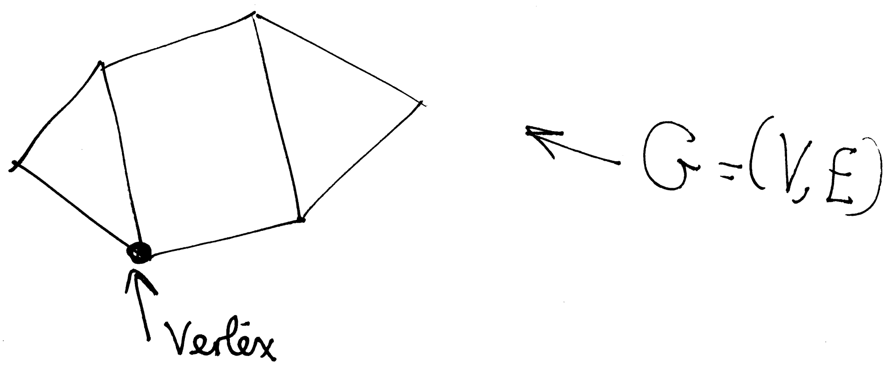
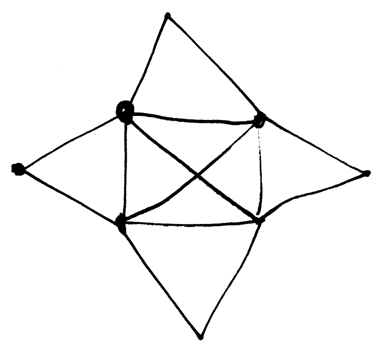
10\16\06

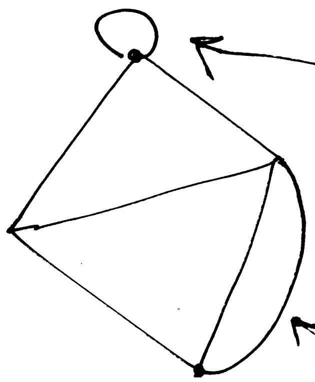
Graph Theory



Eulerian Tour



(Multi)- Graph



A graph is <u>simple</u> if no loops or multiple edges

"Handshaking" Lemma

$$\sum_{v\in V} d_G(v) = 2|E| .$$

# of 1's in row v $= d_G(v)$

$$\begin{array}{cccccc} & & & e & & \\ v & \left[\begin{matrix} 1 & 1 & 1 & & 1 \\ & & & & \\ & & 1 & & \end{matrix}\right. & & & & \left.\right] \end{array}$$

Vertex-Edge incidence matrix.

$\#1$'s in matrix $= 2|E|$ — adding by columns

$= \sum_{v} d_G(v)$ adding by rows

$\square$

vertices of odd degree is even

$$\underbrace{\sum_{v:\,ODD} d_G(v)}_{\text{even}} = \underbrace{2|E|}_{\text{even}} - \underbrace{\sum_{v:\,EVEN} d_G(v)}_{\text{even}}$$

$|ODD|$ is even

$\square$

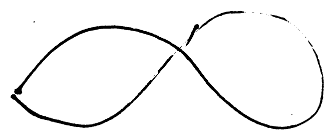
Walk = $v_1, v_2, \ldots v_k$

where (v_i, v_{i+1}) is an edge.

In a walk you can use each vertex more than once.

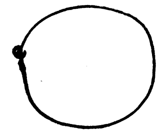
In a <u>path</u> a vertex is used at most once.

<u>Closed</u> walk



$v_1, v_2, \ldots, v_k, v_1$

cycle



$v_1, v_2, \ldots, v_k, v_1$

where $v_1, v_2, \ldots, v_k$ is a <u>path</u>

Adjacency Matrix A

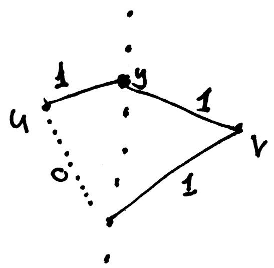
$$\begin{array}{cc} & \begin{matrix} \quad & \quad & v \end{matrix} \\ u & \begin{bmatrix} \quad & \quad & 1 \\ \quad & \quad & \quad \end{bmatrix} \end{array}$$

u reachable from v in 1 step

$$A^2 = ?$$

$$\begin{array}{cc} & \begin{matrix} \quad & \quad & v \end{matrix} \\ u & \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & x \\ \quad & \quad & \quad \end{bmatrix} \end{array}$$

$$x = \sum_y A_{uy} A_{yv}$$



= #walks of length 2 from u to v.

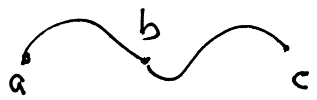
Connectivity

$a \sim b$ iff $\exists$ walk from a to b

$\sim$ is an equivalence relation.

walk from a to a

no orientation $\Rightarrow$ symmetric

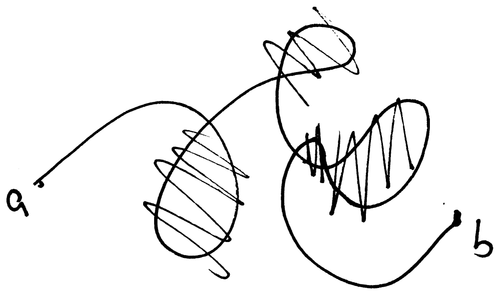


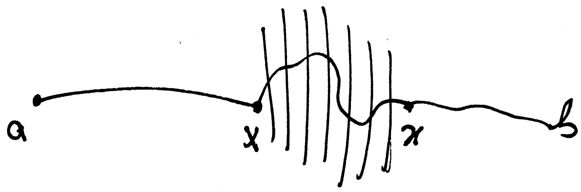
transitivity

Equivalence classes are called <u>components</u>

G is connected if <u>one</u> component.

If there's walk from a to b then there is a path from a to b.





Let P be a shortest walk from a to b.

P will be a path

#edges

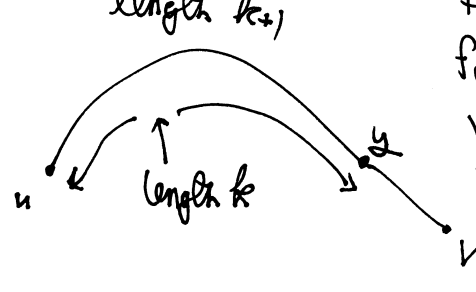
$A^k_{u,v}$ = # of walks of length k from u to v.

By induction: True for $k = 1$

$$A^{k+1}_{u,v} = \sum_y A^k_{u,y} A_{y,v}$$

$A^{k+1} = A^k \times A$

$$= \sum_y \underbrace{\#\text{walks}(u \to y) \times 1_{y \text{ adj. to } v}}_{\#\text{walks of length } k+1 \text{ for which } y \text{ is the } k\text{th vertex}}$$



<u>Walks and not paths</u>