10/06/06

m distinguishable balls, n boxes

ζ = # of empty boxes

$= \zeta_1 + \zeta_2 + \dots + \zeta_n$

$\zeta_i = 1$ if box i is empty.

$$E(\zeta) = n\,E(\zeta_1)$$

$$= n\,\Pr(\text{Box 1 is empty})$$

$$= n \times \frac{(n-1)^m}{n^m}$$

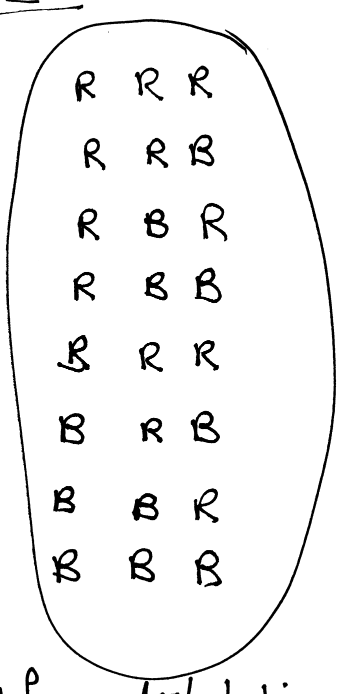
$$= n\left(1 - \frac{1}{n}\right)^m$$

[Slides cover # of non-empty boxes
$= n - \dots$]

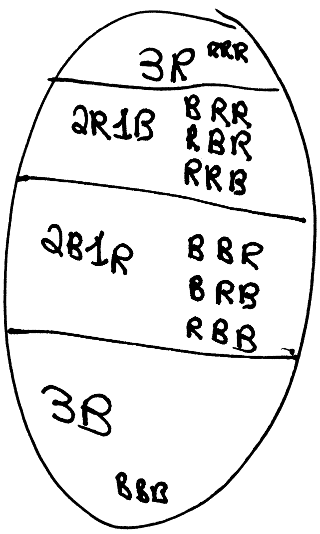
Distinguishable v Indistinguishable

3 balls
2 colors

Ω

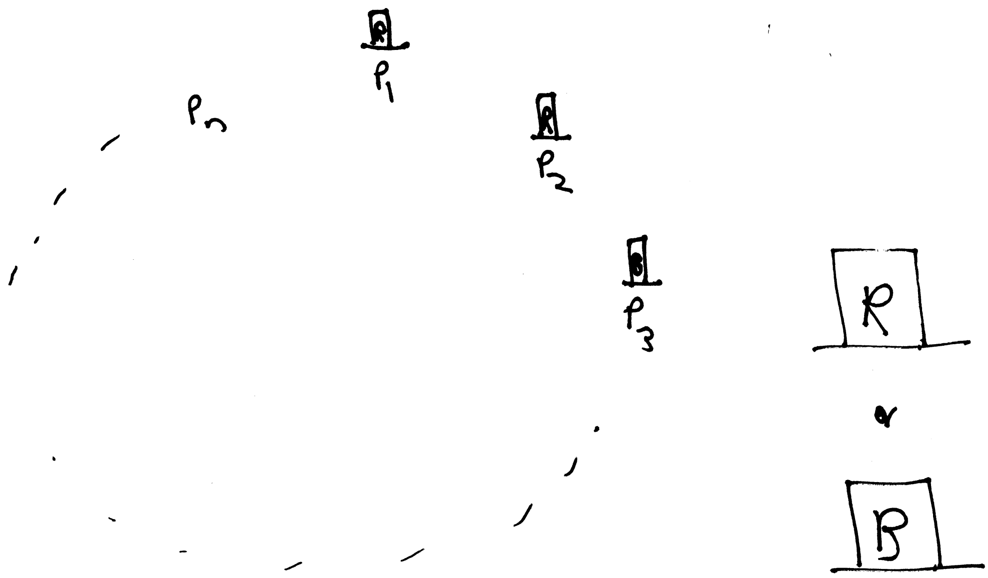


uniform distribution



Uniform over partition

Problem with hats



Every person sees color of every other person, but not own.

Assume hats have a random color.

People win if $\geq$ one person can name color of own hat.

Lose if someone makes a mistake

NO CLOCKS

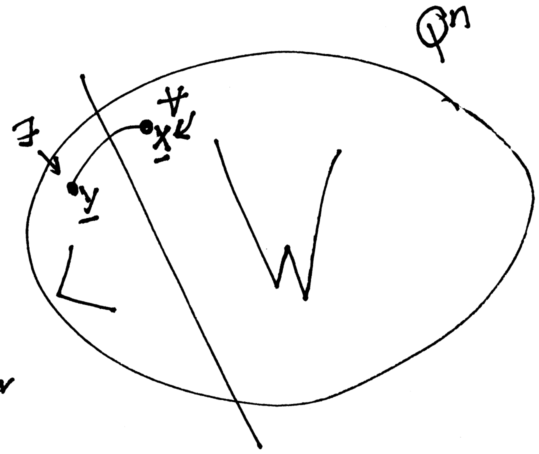
THEY DO HAVE A STRATEGY

Prob. of winning $\geq 1 - \frac{2\ln n}{n}$

Strategy

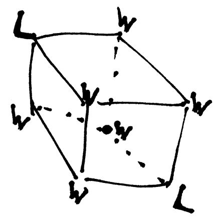
$$Q_n = \{0,1\}^n = L \underset{\text{partition}}{\cup} W$$

L is a cover of

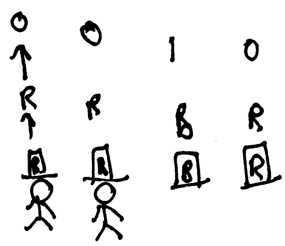


$h(\underline{x}, \underline{y}) = 1$

$\#j : x_j \neq y_j$



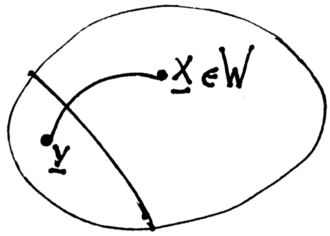
Players agree on L & W.



(i) Enter room, with their hats

(ii) Assume that "hats" $\in W$

(iii) Everybody for which there is a <u>unique</u> hat color placing them in L will announce this color.



CHECK STRATEGY

Person i

0 1 0 0 0 1 ? 0 1 . . . 1

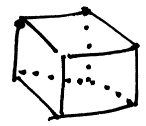
1 2 . . . i n

$\underline{x}$ is one of two possibilities.

$\exists \underline{y}$

$\underline{x} \longrightarrow \underline{y}$ changes bit j

Person j will speak



P1 P2

1 0

L = {0,0,0 & 1,1,1}

W = REST

1 P3

P3 1 0 $\overset{0}{1}$ says nothing

P2 1 $\overset{0}{1}$ 1 says 0.

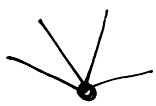
$$p = \frac{\ln n}{n}$$

Choose L_1 randomly; put $y \in Q_n$ into L_1 with probability p.

$L_2 = \{z : L_1$ does not cover $z\}$.

$$L = L_1 \cup L_2$$

$$E(|L|) = E(|L_1|) + E(|L_2|)$$



$$= 2^n p + 2^n(1-p)^{n+1}$$

$$\leq 2^n p + 2^n e^{-np}$$

$$= 2^n \cdot \frac{\ln n}{n} + 2^n \cdot \frac{\ln n}{n}$$

$$= |Q_n| \cdot \frac{2\ln n}{n}$$

$\Rightarrow \exists$ cover of size $\leq |Q_n| \cdot \frac{2\ln n}{n}$.