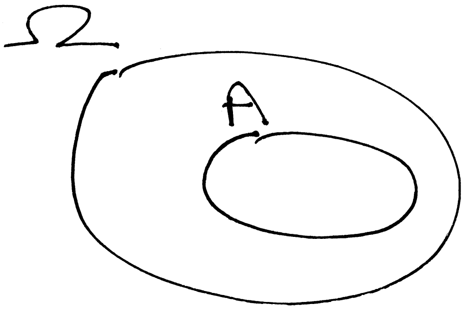
10/02/06

Conditional Probablity



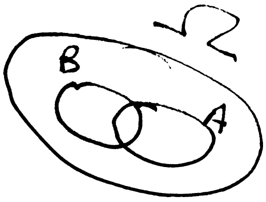
$P(A) > 0$

A *induces* a new probability space.

$$\omega \in A, \quad P_A(\omega) = \frac{P(\omega)}{P(A)}$$

So $$\sum_{\omega \in A} P_A(\omega) = 1.$$

$B \subseteq \Omega$ is an event.



$$Pr(B|A) = \frac{Pr(B \cap A)}{Pr(A)}$$

Two dices (x_1, x_2) $\qquad |\Omega| = 36$

$$A = \{x_1 = 3\}$$

$$B = \{X_1 + X_2 > 6\}$$

$$\boxed{\begin{aligned} Pr(B) &= 1 - P(X_1 + X_2 \leq 5) \\ &= 1 - \frac{1}{36}(1+2+3+4) \\ &= \frac{13}{18} > \frac{1}{2} \end{aligned}}$$

$$P(B|A) = \frac{Pr(B \cap A)}{Pr(A) = 1/6}$$

$$P(X_2 > 3) = \frac{1}{2}$$

$$= \frac{1}{2}.$$

$$\boxed{P(B|A) = P(A) \text{ iff } A, B \text{ are independent}}$$

Boy Girl Problem

Family : 2 children. $X > Y$
$\uparrow$ eldest

$P(\text{Two boys} \mid \text{at least one boy})$.

A BB $\frac{1}{4}$	A BG $\frac{1}{4}$
A GB $\frac{1}{4}$	GG $\frac{1}{4}$

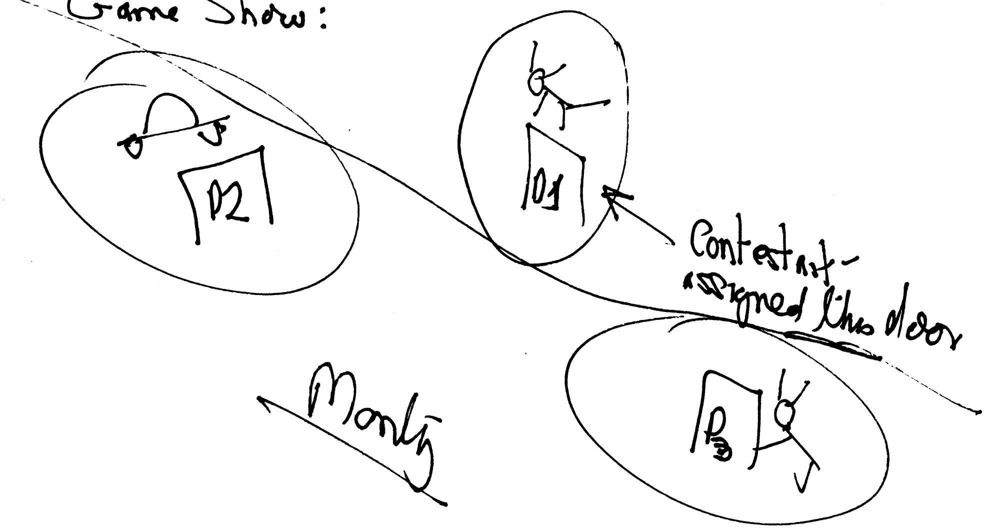
Ω

$P(A) = 3/4$

$$P(BB \mid A) = \frac{P(BB \wedge A)}{P(A)} = \frac{P(BB)}{P(A)} = \frac{1}{3}$$

Monty Hall Paradox

Game Show:

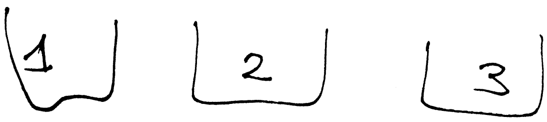


Monty opens door 2 or 3 to show a goat.

Contestant can switch doors.

Question: Is there any advantage??

Incorrect Analysis

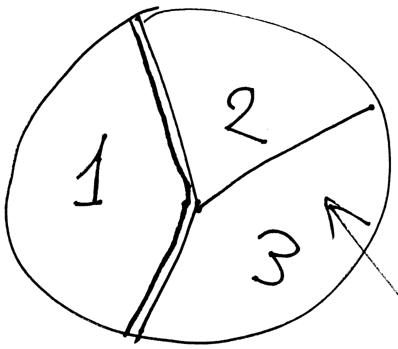


$i \equiv$ car behind i,

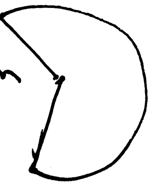
$$P(\underset{\text{chosen door wins}}{1} \mid \underbrace{\text{not } b}_{\text{Monty tells you}}) = \frac{P(1)}{P(\neg b)} = \frac{1/3}{2/3} = \frac{1}{2}.$$

Marilyn Savant published article claiming you ought to change.
Set off a firestorm of criticism

Correct Analysis



$\frac{2}{3}$ chance car is here

Monty allows contestant to choose from  in one go.

Chance of winning by switching is $2/3$.

$a = 1$ [Contestant's choice]

$$\Omega = \{\overset{1/6}{12}, \overset{1/6}{13}, \overset{1/3}{23}, \overset{1/3}{32}\}$$

Car is behind 1 and host opens 2

22 & 33 are missing

$P(1) = 1/3$.

Binomial

toss n coins

$$\Omega = \{ \underbrace{HTTHH\ldots\ldots H}_{n} \}$$

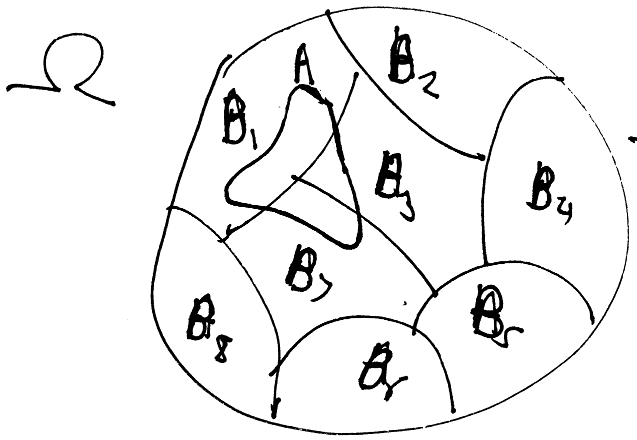
$$\Pr(H) = p$$

$$A \subseteq [n] \qquad |A| = k$$

$$\Pr(A = \text{place where you get heads} \mid k \text{ heads}).$$

$$= \frac{\Pr(A = \{H's\}) = p^k(1-p)^{n-k}}{\Pr(k \text{ heads}) = \binom{n}{k} p^k (1-p)^{n-k}} = \frac{1}{\binom{n}{k}}$$

Law of total probability



Simplifies probability calculation

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \cdots + P(A \cap B_n)$$

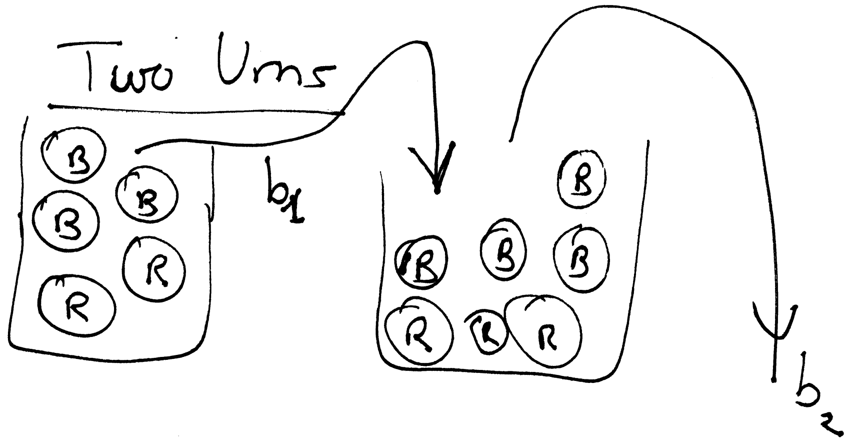
$\boxed{A \cap B_i,\ A \cap B_j \text{ are disjoint}}$

$$= P(A|B_1)\,P(B_1) + \cdots$$

$$P(A \cap B) = P(A|B) * P(B)$$

$$\frac{P(A \cap B)}{P(B)}$$

Two Urns



What $P(b_2=B)$?

$$P(b_2=B) = \Pr(b_2=B \mid \overset{5/8}{} b_1=B)\,\overset{3/5}{P(b_1=B)}$$

$$+ \Pr(b_2=B \mid \underset{1/2}{} b_1=R)\,\underset{2/5}{P(b_1=R)}$$