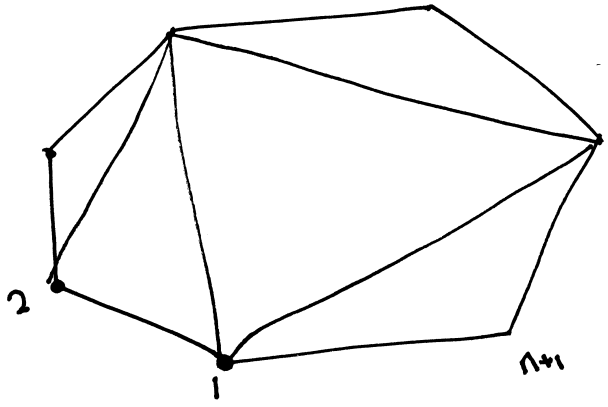
09/18/06

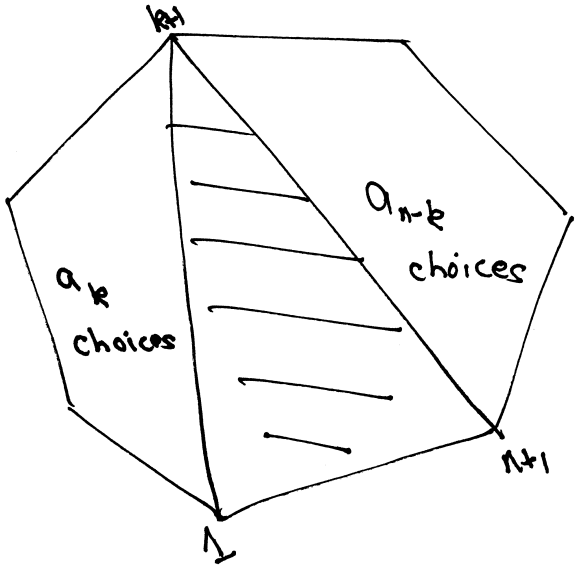


a_k = # ways of triangulating polygon with $n+1$ sides

$$= \sum_{k=0}^{n} a_k a_{n-k}$$

$a_0 = 0$

$a_1 = a_2 = 1$



multiply each equation by x^n and sum:

$$\sum_{n=2}^{\infty} a_n x^n = \sum_{n=2}^{\infty} \left(\sum_{k=0}^{n} a_k a_{n-k} \right) x^n$$

$$a(x) - X = a(x)^2$$

$$\sum_{k=0}^{n} a_k a_{n-k} = 0 \quad \text{if } n = 0 \text{ or } 1$$

or

$$a(x)^2 - a(x) + X = 0$$

$$a(x) = \frac{1 \pm \sqrt{1 - 4x}}{2}$$

$a(0) = 0 \Rightarrow$

$$a(x) = \frac{1 - \sqrt{1 - 4x}}{2}$$

$$= \frac{1}{2}\left(1 - \sum_{n=0}^{\infty} \binom{1/2}{n} (-4)^n x^n \right)$$

or

$$a_n = -\frac{1}{2} \binom{1/2}{n} (-4)^n \quad \text{for } n \geq 1$$